1. **Answer:** 250. To reach 90% in the least number of problems involves Jim getting everything correct. Let x be the number of questions he needs to do. Then $\frac{20+x}{50+x}=\frac{9}{10}$ and cross multiplying and solving yields the answer.
2. **Answer:** 25. Composing the two functions by plugging in the function for x in the other expression, we find g(f(x))=6x-12 and f(g(x))=6x-37. Subtracting, we get the answer
3. **Answer:** 3. Factor a p^3 out of the expression to leave p+5. We now need p+5 to be a perfect cube. Clearly the first time this occurs in when p=3. Realize that a perfect cube multiplied by a non-cube cannot be a perfect cube.
4. **Answer:** 4. There are only four cases to consider. Michael is a duck and Ben is a goose, Michael is a goose and Ben is a duck, and both are the same and Eugene is a duck. Then we use the other statements to find a contradiction in one of those. Case 1 reaches an immediate contradiction, because Michael's statement is a contradiction as he, a duck, correctly says Ban is a goose. Case 2 reaches a similar contradiction. The third case, if Ben and Michael are both geese and Eugene is a duck, a contradiction as Chang, a duck, correctly says Eugene is a duck. The final case is that Ben and Michael are both ducks and Eugene is also a duck. This case does not reach a contradiction, and Chang is the only goose in the group.
5. **Answer:** 42. Subtracting the given fractions from 1, we get $\frac{1}{6}$ of the time is equal to 7 hours. Therefore, the period is 42 hours.
6. **Answer:** $\frac{1}{3}$. Harry can place the e before, in between, and after the two numbers. Only one of the three choices is correct.
7. **Answer:** 50 miles. The trick to this question is that Christine runs the whole time Mr. and Mrs. Smith are walking. It takes the couple 2 hours to reach New York. So multiply Christine's speed by the time to get the distance.
8. **Answer:** 7. The units digit of powers of 7 repeats once every 4 powers. 7^0=1, 7^1=7, 7^2=4**9**, 7^3=34**3** and starts repeating with 7^4. Raising a power by a power, we multiply the exponents and find the units digit of 7^{45}. In the cycle, 45 is clearly the 2nd one, so the last digit must be 7.
9. **Answer:** $\frac{1}{4}$. There are now only 12 available spots on the Mu Team, with 16 people up for consideration. 4 people are left off the team. Let Josh be one of them. Out of 15, we choose 3 others to be left out as well. The total number of different selections is 16 choose 12. The probability Josh is left off the team is therefore $\frac{\binom{15}{3}}{\binom{16}{12}} = \frac{1}{4}$
10. **Answer:** 24. The total distance for the trip is 60 miles, and the total time is two and a half hours. Divide to find the speed.
11. **Answer:** 4. The opponents of Manchester must have scored in at least 7 of their 8 games. We want Manchester to lose by three goals in the games they lose, and to win by just one goal in their wins as to not waste their goals. They lose 3-0 four times. They win 1-0, 3-2, and 4-3 twice. This comes out exactly to 4 wins. Note: there are also other ways to achieve 4 wins, but under no circumstance do they achieve 5 wins, because they would lose by at most 9 goals in the three games, and win by at least one in their five wins. This goal difference does not come out to 8.
12. **Answer:** 38. Basically, Tom needs to get 9 of 10 hard questions correct. He has 10 ways to get a question wrong, and his chances of getting 9 correct out of 10 is $\left(\frac{3}{4}\right)^9 \times \frac{1}{4}$. Multiply everything and express in the given form.

13. **Answer:** 4. The rabbit population is the fourth power of three and the ant population is the fourth power of five. After 4 hours, both will be the fourth power of 15.
14. **Answer:** 49π . The length of the chord from the outer circle to the inner circle is 7. By the Pythagorean Theorem, the difference of the squares of the two radii is 49. Therefore the difference in areas of the two circles is 49π .
15. **Answer:** 6. There are many more powers of 2 in 30! than powers of 5 in 30!. There are 7 powers of five in 30 (don't forget 25 is 5 squared), so 30 ends in 7 zeroes (each 2*5 results in an extra zero). However, the denominator takes away one power of 5 (and not enough powers of 2 to make it less than 6) to leave 6 terminating zeroes.
16. **Answer:** 43. This is a case of the Chinese Remainder Theorem. However, we know in this case that the answer is one more than a multiple of 21. Try 22, 43 etc. and come to the answer. By the aforementioned theorem, there is guaranteed to be one and only one such number less than 105.
17. **Answer:** $\frac{1}{8}$. The only way for Dr. Early to flip a sequence of HHH is to begin with such a sequence. If he flips a T at anytime before HHH occurs, there is no way he can get HHH before THH. The probability of having HHH as the first three flips is $\frac{1}{8}$.
18. **Answer:** 3/7. Consider the case when the two boys are at the ends of the row. The number of such distinguishable arrangements is $\binom{3}{2}\times 5!$. The number of distinguishable arrangements when two girls are sitting at the ends of the row is, similarly, $\binom{4}{2}\times 5!$. <u>Now we also know that these pairs of boys or girls can be switched, so that one is sitting at the left end and one is sitting at the right end and vice versa. So we multiply the number of ways by 2</u>. We divide the sum of these two by the total possible ways to line up the 7 people, namely 7!
19. **Answer:** 2004. Christine draws lines to all but three points: the point she chooses and the two adjacent vertices. The 2003 lines she draws separates the region into 2004 regions (draw this with a polygon with fewer sides). The general answer to the question, if given an n-gon is n-2.
20. **Answer:** $\frac{9}{49}$. Consider where the center of the coin lands. To land on the table, the center must land within a circle of radius 7. To land in the hole, the center must be within a circle of radius 3. Thus the probability is the ratio of the areas of the two circles.
21. **Answer:** 56. Without at least 3 threes, one cannot reach 15. Now the problem is reduced to the number of ways one can score 6 in 3 problems. The three ways to do this are (2,2,2), (3,2,1), and (3,3,0). Now we must find the unique permutations of each of the three ways to achieve 15. The numbers of permutations are 20, 30, and 6 for the three respective cases.
22. **Answer:** 28. Apply the Pythagorean Theorem to the triangle to find the hypotenuse to be 25. Then answer the question by adding up the sides and subtracting in from the area.
23. **Answer:** $50\sqrt{6}$. Using the Pythagorean Theorem, we add the squares of the height and half the diagonal of the base to get the square of any edge that contains E. The fourth root of the product is just the product of one of the sides that contains E and a side of the base.
24. **Answer:** -1. We could just find the two roots using the quadratic equation and plug into the expression. However, to find the sum of the reciprocals of the roots, just reverse all the coefficients and evaluate $-\frac{b}{a}$, where b is the coefficient of the second highest power, and a is the coefficient of the lead term. Either way, we should reach the same answer.

25. **Answer:** $3\sqrt{3}$. If we split the hexagon into 6 equilateral triangles, we can derive the formula of a regular hexagon in terms of the side length $\frac{3\sqrt{3}}{2}s^2$. The side length in this case is just the radius of the circle.
26. **Answer:** 24π Every time we are asked to solve a 3D geometry problem, we want to reduce it to a 2D problem. If we take a cross-section of the ball through the center and the highest point out of the water, we end up with a circle and a chord, the surface of the water. Using the Pythagorean Theorem, we find that half the chord is 12 meters (The other leg is 5 by the given condition about the top of the ball). This represents the radius of the circle whose circumference we are trying to find.
27. **Answer:** 1440. The first watch will tell the correct time after gaining 720 minutes, which takes 720 days. The second watch will tell the correct time after losing 720 minutes or 480 days. The least common multiple of these numbers is 1440 days.
28. **Answer:** 100. When Chang has finished the race, Brian has run 1800 meters and Eugene has run 1710 meters. While Brian runs the last 200 meters, Eugene only runs $200\times\frac{1710}{1800}$ = 190 meters. Thus, as Brian finishes, Eugene has run 1710+190=1900 meters.
29. **Answer:** -5050. We can only get the term x^{99} if we chose an x from 99 of the parentheses and one of the numbers. Therefore the coefficient is -1-2-3…-100.
30. **Answer:** 6561. The probability of Bob being in trouble is just 1 – the probability that he gets exactly 1 date. That probability is 5 times $\left(\frac{9}{10}\right)^4\times\left(\frac{1}{10}\right)$. This comes out to $\frac{13439}{20000}$.
31. **Answer:** 4. Clearly, each face's number is added four times in the big sum, because each face has 4 vertices. No matter what the sum of the numbers of the 6 faces is, the expression is guaranteed to be divisible by 4.
32. **Answer:** 10. We could just draw out the possible paths. The other approach is using Dyck paths. Only $\frac{1}{n+1}$ of the $\binom{2n}{n}$ possible paths don't cross the y=x line either from below or above. Therefore, we may have both cases, so we multiply the number by 2.
33. **Answer:** $\frac{3\sqrt{3}}{4}$ PQRSTU is a regular hexagon because the side lengths are the same and all the angles are congruent. Each side of the hexagon has length $\sqrt{\left(\frac{1}{2}\right)^2+\left(\frac{1}{2}\right)^2}=\frac{1}{\sqrt{2}}$. Thus, the area of the hexagon is 6 times an equilateral triangle with the same side lengths.
34. **Answer:** 17. This expression simply reduces to a quadratic. Since the expression repeats infinitely, we can set the expression to x and then plug x into the equation like so $x=1+\cfrac{1}{2+\cfrac{1}{x}}$.

    Solve for x to obtain $\frac{1+\sqrt{3}}{2}$
35. **Answer:** 5. Plugging in x_1, we get,
    $(1)^2 – (1)(x_2) = (-2)^1$
    $x_2 = 3$
    Now, plugging the known values into the expression involving x_3, we get $9-x_3=4$, and $x_3=5$.

36. **Answer:** 3. Do case analysis for this problem. In the first quadrant, the region is bounded by x + y = 1. In the 4^{th} quadrant, the absolute values will represent the negatives of everything, so the region will be bound by x + y = -1. In the other two quadrants, the x + y absolute value changes sign on opposite sides of the line y=-x. The four lines that are bounds are x=-1, y=1, x=1, and y=-1.

37. **Answer:** 2+ $\sqrt{3}$. The height of the rhombus is 0.5x, where x is the length of each side, because the area of the rhombus must equal half of x^2. Call the vertices A, B, C, and D. Draw altitudes from A and B to base CD (one altitude will not intersect the base, say B, making BD the long diagonal). Call the feet of these altitudes E and F respectively. We know the length of both AE and AD in terms of x, so we can find DE by the Pythagorean Theorem. By congruent triangles AED and BCF, DE=CF. The square of the long diagonal will equal $(BF)^2+(DC+CF)^2$ and the square of the short diagonal is $(DC-DE)^2+AE^2$. Thus,

$$DE = \frac{\sqrt{3}}{2}x$$

$$EC = (1-\frac{\sqrt{3}}{2})x$$

and

$$\frac{BD}{AC} = \frac{\sqrt{\frac{1}{4}x^2 + (1+\frac{\sqrt{3}}{2})^2 * x^2}}{\sqrt{\frac{1}{4}x^2 + (1-\frac{\sqrt{3}}{2})^2 * x^2}}$$

rationalize the denominator in the expression which should eliminate the radical.

38. **Answer:** 56. Let A be the top of the statue, B be the bottom of the statue, C be the point on the ground that is on the line AB, and D be the point where the viewer is. From the 45-45-90 triangle, we find that AC=DC is 50. BC=50 * tan (30). Our answer is just AC - BC.

39. **Answer:** $\frac{4000\sqrt{2}}{3}$. The volume of the pyramid is $\frac{1}{3}\times base \times height$. We already know the area of the base, 20^2, so we need to find the height. The other four faces of the pyramid are equilateral triangles. The height of the triangle is $10\sqrt{3}$. Draw the line from the midpoint of a side of the square base (also the foot of the altitude of an equilateral triangle face) to the center of the square. Then draw a vertical line from the center of the square to the apex (tip of the pyramid). Use the Pythagorean Theorem to find the height and plug into the formula for volume.

40. **Answer:** 3. The ratio we are looking for is just $\frac{1}{2}$. By the angle bisector theorem, MC is $\frac{6\times 7}{12} = \frac{7}{2}$. Looking at triangle ACM, the angle bisector of C intersects AM at I. Once again by the angle bisector theorem, the ratio we are looking for is just CM/AC.

41. **Answer:** $\frac{21}{33}$. The most favorable distribution will be to have 2 boxes with only one white ball and a third box with 6 black and 16 white balls. The least favorable distribution is to have 2 boxes with 1 black ball in each and the third box with 18 white and 4 black balls. In the first case, he has a $\frac{2}{3}$ chance of picking a box with only white balls and a $\frac{1}{3}$ chance of picking the box where he has an $\frac{8}{11}$ chance. His total chance of winning is $\frac{10}{11}$

In the second case he has no chance of winning if he picks 2 of the boxes. He has a $\frac{1}{3}$ chance of picking the box with white balls in, and has $\frac{9}{11}$ chance of picking a white ball. Thus his total chance of winning is just $\frac{9}{33}$. Thus the difference is the answer.

42. **Answer:** $\frac{25}{3}$. The twins will collect their first different toy by buying one box of fries. They have a $\frac{4}{3}$ chance of getting a different character by buying their next boxes of fries. The expected number of boxes they need is $\frac{4}{3}$. Then they have a $\frac{1}{2}$ chance of collecting a third different toy and a $\frac{1}{4}$ chance of collecting a fourth different one. The expected number of turns it takes is therefore $1+\frac{4}{3}+2+4=\frac{25}{3}$.

43. **Answer:** $\frac{7}{9}$. We write out the 6 polynomials that we can have, and pick the three highest values of (sum of roots+ product of roots). The sum of the roots is –b/a and the product of the roots is c/a. The three highest values are 1, 1, and $\frac{1}{3}$. The average of these three values represents the sum of the co-ordinates of the centroid of the triangle (The x and y co-ordinates of the centroid are just the average of the three x co-ordinate and the three y co-ordinate values respectively).

44. **Answer:** $\frac{2}{3}$. Pick a random point on the circumference of the circle. Draw two equilateral triangles, both having the point and the center of the circle as vertices. Now the third vertex of both equilateral triangles represent the extreme points where a chord from the point we chose to another point on the circle will have a longer length than the radius. The arc on the circle where the chord will be longer is 360-120=240. Thus there is a $\frac{2}{3}$ chance the chord will have sufficient length.

45. **Answer:** $\frac{438}{4+\sqrt{3}}$. Line 5 circles up against the edge of a side of the triangle. Extend perpendiculars from the circles on each end, call their centers O and P, to intersect the side at A and B respectively. Side AB has length 8r. The length from A to the nearest vertex is 438-4r. From the fact that the length of the side opposite the 30 degree angle in a right triangle is one half of the hypotenuse and that the line from the vertex, call it V, to O bisects the 60 degree vertex angle, we find the following relation $\frac{OA}{AV}=\frac{r}{438-4r}=\frac{\frac{1}{2}}{\sqrt{1-\left(\frac{1}{2}\right)^2}}=\frac{1}{\sqrt{3}}$. Solving for r, we obtain the answer.

46. **Answer:** 120. New Jersey must win a sixth game. Prior to that, it doesn't matter how the series becomes 3-2 in favor of New Jersey. Therefore the probability that it become 3-2 is

$\binom{5}{3}(\frac{3}{4})^3(\frac{1}{4})^2$. Multiply that by 0.75 for a New Jersey win in the six game, we find the probability. To find the number of factors, we add one to the power of each prime in the prime factorization of a number and multiply those numbers together.

47. **Answer:** $\frac{1}{2}$. The problem with Mr. Holbrook's test, as it is with many medical tests, is that the amount of false positives in the general population equals or exceeds the number of correct positives in the genius population. Thus the answer is a ratio of the probability of being tested correctly as a genius to the probability of being tested as a genius in general. Thus the ratio is $\frac{0.05*0.95}{0.05*0.95+0.95*0.05}=\frac{1}{2}$.

48. **Answer:** 3.

Since

$$1 \le x \le y \le z \le u \le v,$$

$$4+v \le x+y+z+u+v \le 5v$$

$$\therefore 4+v \le xyzuv \le 5v$$

$$\therefore xyzu \le 5$$

$$4+v \le xyzuv$$

$$\therefore 4 \le (xyzu-1)v$$

$$\therefore xyzu > 1$$

$$\therefore 2 \le xyzu \le 5$$

Trying possibilities:

x,y,z,u	Find u so that Sum=Product	Valid solution?
1,1,1,2	v=5	Yes
1,1,1,3	v=3	Yes
1,1,2,2	v=2	Yes
1,1,1,4	v= $\frac{7}{3}$	No
1,1,1,5	v=2	No, v<u

Therefore, there are three solutions.

1. **Answer:** 625. Call the point where the two arcs meet D and the end of one of the roads A. Extend a line through D parallel to the roads and a line through A perpendicular to the road. These lines meet at point E. AE must be half of AC (=1200) and ED must be half of BC (=900), both by symmetry. Now call the center of the arc O. An extension of line segment DE passes through O because the arcs are tangent to each other. We already know that AE is perpendicular to ED because AC is perpendicular CB. Both OD and OA are length r. OE= OA – ED= r – 450. Apply the Pythagorean Theorem to right triangle OEA and solve for r.
2. **Answer:** 49. We can separate this problem into two cases, the one in which the mostly likely person to fall asleep falls asleep and the one in which one of the other 15 fall asleep. The answer is therefore $1\times\frac{3}{4}\times\left(\frac{3}{4}\right)^{15}+15\times\frac{1}{4}\times\frac{1}{4}\times\left(\frac{3}{4}\right)^{14}=\frac{3^{15}}{2^{29}}$