4^{th} Grade Questions and Solutions

Bergen County Academies Math Competition 2006

22 October 2006

1. Brian likes to feed the ducks. The first day he uses 1 slice of bread. The second day he uses 2 slices of bread. The third day he uses 3 slices of bread. If this pattern continues, how many slices of bread will he use on the tenth day?
   Answer: 10
   Follow the pattern: the fourth day he will use 4 slices of bread, and so on, until the tenth day when he will use 10 slices of bread.

2. To get ready for a math competition, Kevin buys a pencil for fifty cents, a ruler for one dollar, and graph paper for thirty dollars. How much money does Kevin spend?
   Answer: \$31.50
   Add the three costs, lining up decimal points:
   $$\begin{array}{r} \$0.50 \\ \$1.00 \\ +\$30.00 \\ \hline \$31.50 \end{array}$$

3. If CNN is on channel 25 and YES is on channel 70, how many times does Michael have to hit the "channel up" button in order to switch from watching CNN to YES?
   Answer: 45
   Find the difference between 70 and 25: $70 - 25 = 45$.

4. Chang is illegally cruising down Route 4 in his father's car. There are numbered markers along the route, each spaced one mile apart, which indicate how far one has travelled. If Chang starts at the 94^{th} mile marker and the police stop him at the 117^{th} mile marker, how many miles did Chang travel?
   Answer: 23
   Find the difference between numbers on the mile makers: $117 - 94 = 23$ miles.

5. $2 \times 0 \times 0 \times 6 =$
   Answer: 0
   Anything multiplied by 0 equals 0; thus, since there is at least one zero, the answer is 0.

6. Mrs. Stone makes 20 telephone calls per day. How many telephone calls does she make in the month of January?
   Answer: 620
   There are 31 days in January, and she makes 20 calls on each of these days, so she makes a total of $31 \cdot 20 = 620$ phone calls in January.

7. Arthur accidentally added 7 to a number instead of multiplying that number by 7 on a math problem. His answer was 11. What should he have gotten as an answer?
   Answer: 28

We can work backwards to find the answer. Since he added, we can reverse that by subtracting: the original number was $11 - 7 = 4$. He was supposed to multiply this by 7, and $7 \cdot 4 = 28$.

8. Calculate $10 - 9 + 8 - 7 + 6 - 5 + 4 - 3 + 2 - 1$.
Answer: 5
There are two approaches to this problem. We could simply carry out each addition and subtraction to find the answer. Or we could group the numbers in a clever way that simplifies our computations, namely $(10 - 9) + (8 - 7) + (6 - 5) + (4 - 3) + (2 - 1)$. Following order of operations, we evaluate the expressions in parentheses first and find that the value of each is 1. So this equals $1 + 1 + 1 + 1 + 1$, which is 5.

9. Asena and Christine are playing a game. Asena asks Christine to pick a number. She then asks her to add 12 to that number, multiply her answer by 3, subtract 36 from her new answer, divide that answer by 6, and multiply this answer by 2. If Christine picks the number 10 in the beginning, what number does she have at the end?
Answer: 10
$10 + 12 = 22 \cdot 3 = 66 - 36 = 30 \div 6 = 5 \cdot 2 = 10$.

10. Compute the number of letters in the alphabet plus the number of days in a week.
Answer: 33
There are 26 letters in the alphabet and 7 days in a week. $26 + 7 = 33$.

11. To be prepared, Yoonjoo wants to carry 10 pencils with her at all times during school. Every hour, she gives one pencil to a friend and loses one pencil. If school is 8 hours long, at least how many pencils must Yoonjoo bring to school to remain prepared?
Answer: 26
The first hour, she gives away a pencil and loses a pencil, which means that she is short 2 pencils. Since this occurs every hour, this means that at the end of the day she will be short 16 pencils. Thus she must bring at least $10 + 16 = 26$ pencils to school.

12. James's pet alligator has a body and a tail that together are 5 times as long as its head. From the tip of its nose to the tip of its tail the giant alligator is 300 meters long. How many meters long is its head?
Answer: 50
Since the body and tail are 5 times as long as the head, there are really 6 equal sections of the body that are the all the length of the head. So the head is $300 \div 6 = 50$ meters long.

13. Aaron knows he has somewhere between 100 and 110 baseball cards in his collection. If he counts his cards 2 at a time, he has 1 card left over. If he counts his cards 5 at a time, he has 2 left over. How many cards are in Aaron's collection?
Answer: 107
Since Aaron has one card left over when he counts his cards 2 at a time, we know that he has an odd number of cards. We also know that he has 2 cards left over when he counts his cards 5 at a time. There are only two numbers between 100 and 110 that satisfy this last requirement: 102 and 107. However, we know that the number cannot be even, so Aaron must have 107 baseball cards in his collection.

14. Express $\frac{11}{5}$ in decimal form.
Answer: 2.2
Either divide directly: $11 \div 5 = 2.2$, or note that $\frac{11}{5} = \frac{10+1}{5} = 2 + \frac{1}{5} = 2 + \frac{2}{10} = 2.2$.

15. Mr. Bank Clerk, Mr. Police Officer, and Mr. Teacher work as a bank clerk, a police officer, and a teacher. However, none of them have the job that shares their name. If Mr. Police Officer's wife Mrs. Police Officer is the teacher's twin sister, who is the police officer?
Answer: Mr. Teacher
Since Mr. Police Officer's wife is the teacher's twin sister, Mr. Police Officer is not the teacher. He also cannot be the police officer, so he must be the bank clerk. Since Mr. Teacher cannot be the teacher or the bank clerk, he must be the police officer.

16. In Eddy's sock drawer, there are two red socks, three blue socks, and two green socks. If Eddy takes socks out of the drawer one at a time and does not put them back, how many must he take out so that he can make at least one matching pair?
Answer: 4
Suppose Eddy takes out three socks, and does not yet have a matching pair. This can only happen if he takes out one red, one blue, and one green sock. The next (fourth) sock he takes out must then match one of the socks he has already taken out. If the first three socks were not all different colors, they must have contained a matching pair.

17. The product of two different positive whole numbers is 11. What is their sum?
Answer: 12
Since 11 is prime, the only two positive whole numbers that we could multiply to get 11 are 11 and 1. $11 + 1 = 12$.

18. A military clock, instead of having the numbers 1 through 12, has the numbers 1 through 24 evenly spaced around the circular face. What is the number directly across from 19?
Answer: 7
We could simply draw the clock and match the numbers up. We could also note that numbers opposite each other will always have a difference of 12. So the number across from 19 is $19 - 12 = 7$.

19. In a high school physical education class, 11 students know how to climb a tree. 6 of these students are male. There are 14 boys total in the class. 7 girls in the class do not know how to climb a tree. How many students are there in this class?
Answer: 26
If 11 of the students in the class know how to climb a tree, and 6 are boys, that means that the other 5 are girls. 7 girls in the class do not know how to climb a tree, so there are $5 + 7 = 12$ girls in the class. Therefore the total number of students in the class is 12 girls + 14 boys = 26.

20. For a class trip, Mr. Holbrook hires 2 buses, which can each hold 50 students. However, 123 students signed up for the trip. If a car can hold up to 4 students, what is the minimum number of cars needed in addition to the two buses to ensure that every student has a ride?
Answer: 6
The two buses hold $50 * 2 = 100$ students, leaving $123 - 100 = 23$ students for the cars. Each car holds a maximum 4 people so the 23 kids need at least $23 \div 4 = 5.75$ cars. Since cars are whole, this means that 6 cars are needed.

21. Mike arrived at a party 27 minutes before 3 : 23 PM. What time was it 11 minutes after he arrived?
Answer: 3:07 PM
27 minutes before 3 : 23 is 2 : 56. 11 minutes after 2 : 56 is 3 : 07.

22. If Eugene counted to 600 by 6's, starting with 6, how many numbers did he count that are less than 600?

Answer: 99
Eugene counted $600 \div 6 = 100$ numbers, all of which were less than 600 except for one, namely 600. So he counted $100 - 1 = 99$ numbers that are less than 600.

23. $20 + 40 + 60 + 80 = (1 + 2 + 3 + 4) \times \_?\_$
Answer: 20
Each term on the left hand side of the equation is divisible by 20. Factoring out a 20 yields the expression $(1 + 2 + 3 + 4) \times 20$.

24. If a bird's wings flap 64 times every second, then how many times do a bird's wings flap in 10 minutes?
Answer: 38,400
There are 60 seconds in one minute, so there are $10 \cdot 60 = 600$ seconds in 10 minutes. Thus the bird flaps its wings $600 \cdot 64 = 38400$ times in 10 minutes.

25. If a race began at 3 : 43 PM and ended at 5 : 57 PM the same day, the race was half over at what time?
Answer: 4:50 PM
The entire race was 2 hours and 14 minutes long. So the race was half over at the start time - 3 : 43 PM - plus 1 hour and 7 minutes, or 4 : 50 PM.

26. Hannah goes to Boston Market to eat lunch. She picks a piece of chicken for her main meal, and must select a side dish and a drink. The possible side dishes are mashed potatos, corn, green beans, and cinnamon apples. The possible drinks are Coca Cola, Sprite, orange soda, iced tea, lemonade, and water. How many different meals can Hannah select?
Answer: 24
For each of the four side dishes, Hannah can pick any of the six drinks. So there are $4 * 6 = 24$ total combinations.

27. Dan likes bowling, pool, and math. He decides to number the 10 bowling pins 1 through 10 and adds up the numbers on each pin, obtaining a number P. Then he numbers each of the 15 pool balls 1 though 15 and adds these up to obtain a number B. Find $B - P$.
Answer: 65
We could find B and P separately by adding the numbers 1 through 10 and 1 through 15, and then we could find $B - P$. It is quicker to note that B is just $P + 11 + 12 + 13 + 14 + 15$ (since $P = 1 + 2 + \cdots + 10$). Therefore, $B - P = 11 + 12 + 13 + 14 + 15 = 65$.

28. What is the remainder when 6668 is divided by 6?
Answer: 2
We know that 6666 is exactly divisible by 6, so 6668 is 2 more than a multiple of 6, and will have a remainder of 2 when divided by 6.

29. Every time Joey visits Monica and Chandler's apartment, he steals one more food item than he did the last time. If he visits once every day for a week, and steals 3 items the first day, how many does he steal in the week?
Answer: 42
$3 + 4 + 5 + 6 + 7 + 8 + 9 = 42$.

30. Ashley's calculator only performs one process: when she enters a number, the calculator shifts the entire number one decimal place to the right, adds 11, and returns the resulting number. For example, if she enters 11, the calculator shifts 11 a decimal place to the right to give 1.1, then adds 11 which gives 12.1. If she gets back 32.5 as an answer, what was her original number?

Answer: 215
We reverse the calculator's process: $32.5 - 11 = 21.5$. Shifted one decimal place to the left, this is 215.

31. Caroline chooses a whole number greater than 1. She divides $1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot 7$ by that whole number. Then she divides the quotient by the same whole number. She continues dividing each new quotient by the same whole number until the remainder is no longer 0. Which whole number should Caroline choose so that she performs the division the greatest number of times?
Answer: 2
If we break the composite numbers in $1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot 7$ into prime facotrs, we see that there are four 2's in the prime factorization of this number, but only at most two factors of any other number. Thus Caroline can divide by 2 the greatest number of times.

32. Dr. Mayers raises chickens and rabbits in his classroom. He counts 45 total animals, and 144 legs (each chicken has 2 legs, while each rabbit has 4 legs). How many rabbits does Dr. Mayers have?
Answer: 27
Suppose all the animals are rabbits. Then there would be $4 \cdot 45 = 180$ legs. If we substitute a chicken for a rabbit, we reduce the number of legs by 2. We want to reduce the number of legs by $180 - 144 = 36$, so we must substitute in $36 \div 2 = 18$ chickens. This means there are $45 - 18 = 27$ rabbits.

33. Ben, Joe, Rob and Mike like to play ping-pong after math team. Each played exactly 3 games of ping-pong with each of the others. How many games of ping-pong were played altogether?
Answer: 18
If we label the boys B, J, R, and M, we know that there are 6 games if they each played each other once: BJ, BR, BM, JR, JM, RM. But since they each played 3 games against each other, they played $6 * 3 = 18$ games total.

34. Sally has 7 coins, which together make 72 cents. Each coin is a penny, nickel, dime or quarter. How many dimes does Sally have?
Answer: 1
Sally must have 2 pennies, since every other coin has a cent value that is a multiple of 5. If her other 5 coins are all dimes, she would only have 52 cents, so she must have at least one quarter. If her other 4 coins are all dimes, she would only have $25 + 40 + 2 = 67$ cents, so she must have at least two quarters. If her other 3 coins are all dimes, she would have $50 + 30 + 2 = 82$ cents, which is too much, so at least one coin must be a nickel. This would yield $50 + 20 + 5 + 2 = 77$ cents, which is still too much, so two coins must be nickels. Thus she has 2 pennies, 2 quarters, 2 nickels, and 1 dime ($2 \cdot 1 + 2 \cdot 25 + 2 \cdot 5 + 1 \cdot 10 = 72$).

35. Counting from either end of the line, Michelle is 18^{th} in line. How many people are in the line?
Answer: 35
There are 17 people in front of Michelle and 17 people behind her. Including her, 17+17+1 = 35.

36. If there are 5 zounds to a zong, 7 zongs to a zig, and 3 zigs to a zap, how many zounds are there to 3 zaps?
Answer: 315
There are $5 \cdot 7 = 35$ zounds to a zig, and since there are 3 zigs to a zap, there are $35 \cdot 3 = 105$ zounds to a zap. So there are $3 \cdot 105 = 315$ zounds to 3 zaps.

37. Joey has auditioned for 15 roles in the past two months. He gets 2 out of every 5 roles he auditions for. How many parts does Joey get of the 15?

Answer: 6
$\frac{2}{5} \cdot 15 = 6$.

38. A triangle has a height of length 4 and a base of length 3. What is the triangle's area?
Answer: 6
The area of a triangle is equal to half of the product of its base and height. So the triangle has area $\frac{3 \cdot 4}{2} = 6$.

39. Find the product of the even multiples of 5 that are greater than 1 and less than 49.
Answer: 240000
The even multiples of 5 between 1 and 49 are simply the multiples of 10: 10, 20, 30, and 40. $10 \cdot 20 \cdot 30 \cdot 40 = 240,000$.

40. Connie took a test with 25 questions. For every question she got right, she earned 4 points, and for every question she got wrong, she lost 1 point. She answered every question, and got a score of 80. How many questions did she get right?
Answer: 21
Guess-and-check can be used to solve this problem. Alternately, let c be the number of questions Connie got right. Then she got $25-c$ questions wrong. So her score is $4c-(25-c) = 5c-25 = 80$. Solve for c: $5c = 105 \Rightarrow c = 21$.

41. Rachel begins writing the non-negative whole numbers (0, 1, 2, ...) across a sheet of paper in increasing order, with no spaces between them. She stops when she reaches the end of the paper. There is only room across the paper for 100 digits. What is the last digit Rachel writes?
Answer: 4
The numbers $0-9$ take up 10 spaces. Since there are 90 spaces left and we know that the next set of numbers consist of two digits each, we will be able to fit 45 two-digit numbers, beginning with 10. The 45th two-digit number is $10 + 45 - 1 = 54$, so the last digit in the row is 4.

42. Sixteen teams compete in a soccer tournament. Each game, one team wins and one team loses, and the losing team is eliminated. How many games must be played so that only one team remains undefeated?
Answer: 15
Fifteen teams must be eliminated, and it takes one game to eliminate one team, so 15 games must be played.

43. A square has an area of 16 square inches. What is the length, in inches, of one of its sides?
Answer: 4
The area of a square with side length s is $s \cdot s$. Since $4 \cdot 4 = 16$, a square with area 16 square inches has side length 4 inches.

44. A can of mango juice concentrate is mixed with 3 cans of water to make $1\frac{1}{2}$ pints of mango juice. Each can of water is the same size as the can of mango juice. What fraction of a pint does one can hold?
Answer: $\frac{3}{8}$
If 4 cans can hold $1\frac{1}{2}$ pints, which is the same as $\frac{3}{2}$ pints, then 1 can can hold $\frac{1}{4} \cdot \frac{3}{2} = \frac{3}{8}$ of a pint.

45. Adrian is working at Six Flags dressed up as Tweety Bird. He poses for pictures with 40 people. 35 of these people are female and 15 are children. How many of the 40 are female children?
Answer: 10
Add the number of females and the number of children, and subtract the total number of people: $(35 + 15) - 40 = 10$. This uses the principle of *inclusion-exclusion*.

46. What is the number halfway between $\frac{1}{13}$ and $\frac{1}{9}$?
Answer: $\frac{11}{117}$
Find the arithmetic mean of the two fractions. Add the two fractions: $\frac{1}{9} + \frac{1}{13} = \frac{13+9}{9 \cdot 13} = \frac{22}{117}$. Divide this sum by 2 to yield $\frac{11}{117}$.

47. Sujin paints $\frac{1}{4}$ of a pole green. Christine then paints $\frac{2}{3}$ of the pole red, and leaves the remaining 2 feet unpainted. How many feet long is the entire pole?
Answer: 24
$\frac{1}{4} + \frac{2}{3} = \frac{3}{12} + \frac{8}{12} = \frac{11}{12}$. The remaining$1 - \frac{11}{12} = \frac{1}{12}$ of the pole is 2 feet long; thus the entire pole is $2 \cdot 12 = 24$ feet long.

48. The sum of five different positive integers is 100. What is the largest possible value for any one of these integers?
Answer: 90
In order to find the largest number, the other four numbers must be as small as possible. The four smallest distinct positive integers are $1, 2, 3$ and 4. Thus the largest possible value for one of the integers is $100 - (1 + 2 + 3 + 4) = 100 - 10 = 90$.

49. A *palindrome* is a number that reads the same left to right as it does right to left. For example, 123321 is a palindrome. How many palindromes are there greater than or equal to 1 and less than 100?
Answer: 18
Every one-digit number reads the same forwards and backwards. A two-digit number reads the same forwards and backwards if and only if both of its digits are the same. So the palindromes between 1 and 100 are the 18 numbers $1, 2, \ldots, 9, 11, 22, \ldots, 99$.

50. The average (arithmetic mean) of five numbers in a list is 35. The average of the first two numbers is 26. What is the average of the last three numbers?
Answer: 41
The sum of all the numbers in the list is $5 \cdot 35 = 175$. The sum of the first two numbers is $2 \cdot 26 = 52$, so the sum of the last three numbers is $175 - 52 = 123$. The average of these numbers is $123 \div 3 = 41$.