7th Grade Competition Solutions

Bergen County Academies Math Competition
19 October 2008

1. Before taking the USAMO, a student notices that he has a bag of Doritos, two bags of Fritos, a can of soda, two bags of Skittles, two bags of Cheetos, and three Reese's cups. Each bag of Doritos has 130 calories, each bag of Fritos has 120 calories, each can of soda has 140 calories, each bag of Skittles has 60 calories, each Reese's cup has 30 calories, and each bag of Cheetos has 110 calories. If, during the test, the student consumes everything on his desk, how many calories will he have consumed?
Answer: 940
$1 \cdot 130 + 2 \cdot 120 + 1 \cdot 140 + 2 \cdot 60 + 2 \cdot 110 + 3 \cdot 30 = 130 + 240 + 140 + 120 + 220 + 90 = 940$.

2. $7 + 77 + 777 + 7777 + 77777 = ?$
Answer: 86415
$7 + 77 + 777 + 7777 + 77777 = 7(1 + 11 + 111 + 1111 + 11111) = 7 \cdot 12345 = 86415$.

3. Find the perimeter of an equilateral triangle with side length 9.
Answer: 27
The perimeter of a triangle is equal to the sum of the lengths of its sides, or $3 \cdot 9 = 27$.

4. Josh and Kun-Soo have 2008 Frosted Flakes each. Josh gives Kun-Soo half of his flakes. After this, Kun-Soo gives all of his flakes to Josh. How many flakes does Josh have now?
Answer: 4016
Since the last exchange involves Kun-Soo giving all of his flakes to Josh, Josh now has $2 \cdot 2008 = 4016$ Frosted Flakes.

5. If each minute of music takes up .9 megabytes of memory, then how many megabytes will a library containing 1440 minutes of music take up?
Answer: 1296
The library will take up $1440 \cdot .9 = 1296$ megabytes.

6. David Rush is in a hurry to get to class. His dorm at Philips Exeter is 300 meters away from his first class. He runs the first half of the distance at a brisk 5 meters per second. After this, he (instantly) slows down to 2 meters per second and finishes running at this rate. How many seconds did it take him to get to class?
Answer: 105
The first half of the distance (150 meters) takes David $\frac{150}{5} = 30$ seconds. The second half of the distance takes him $\frac{150}{2} = 75$ seconds. So, it takes David $30 + 75 = 105$ seconds to get to class.

7. A palindrome is a number that is read the same way forwards and backwards. Find the next largest palindrome after 80908.
Answer: 81018
Normally, to find the next highest palindrome, we add one to its center digit(s). In this case, the center digit is as high as it can be, so we must add one to the pair of digits that are the second-closest to center, and decrease the center digit to 0.

8. Using the approximation 1 mile equals 1.6 kilometers, how many miles is 80.8 kilometers equivalent to?
Answer: 50.5
80.8 kilometers$\cdot 1 = \frac{80.8 km}{1} \cdot \frac{1 mi}{1.6 km} = \frac{80.8}{1.6} = 50.5$ miles.

9. Starting with the letter D, a large crowd of students repeatedly chants the letters D, B, and R in that order. Assuming the chant lasts long enough, which letter will be the 2008^{th} letter chanted by the students?

Answer: D
2008 leaves a remainder of 1 when divided by 3. Thus, the sequence of letters will be repeated a large number of times until there is only one letter left to be said. That letter is the first one in the sequence, which is D.

10. Evaluate $(1^2-0^2)+(2^2-1^2)+(3^2-2^2)+(4^2-3^2)+\ldots+(10^2-9^2)$.
Answer: 100
The 1^2 in the first term cancels with the one in the second term; the same goes for 2^2, 3^2, and so on, and all we are left with is $10^2=100$.

11. Matt's suitcase weighs too much to put on the airplane – 56 pounds and 3 ounces. If the maximum allowable weight for a suitcase is 50 pounds, and one pound equals 16 ounces, then what is the minimum number of ounces of luggage Matt must remove from his suitcase before flying?
Answer: 99
Matt must remove 6 pounds and 3 ounces from his suitcase. This weight is the same as $6\cdot16+3=96+3=99$ ounces.

12. Solve for x: $|x-|3x||=6$.
Answer: 3 and –1.5
The first case is $x-|3x|=6$. If $x\geq0$, this is $-2x=6\Rightarrow x=-2$, which is impossible since we said $x\geq0$. If $x<0$, the expression simplifies to $x-(-3x)=6\Rightarrow 4x=6\Rightarrow x=1.5$, which is also a contradiction. The second case is $x-|3x|=-6$. If $x\geq0$, this is $-2x=-6\Rightarrow x=3$. If $x<0$, this is $x-(-3x)=-6\Rightarrow 4x=-6\Rightarrow x=-1.5$, and our solutions are 3 and -1.5.

13. Hannah is cramming for the SAT, and needs to memorize 300 words. She can cram 50 words each morning, but will forget 40 of them after the next night. After how many mornings of studying will Hannah be able to write the correct definitions of all 300 words?
Answer: 22
After two mornings, Hannah will have memorized 100 words. Each morning hereafter, Hannah will know $50-40=10$ more words than she knew after the morning before. Thus, it will take Hannah $2+\dfrac{200}{10}=22$ mornings of studying to learn all of the words.

14. A cylinder with radius 8 and height 17 is rolled along the floor so that it completes 2 full revolutions. What is the area of the region of the floor that was touched by the cylinder during the roll?
Answer: 544π
This region is a rectangle of width 17 and length twice the circumference of the base of the cylinder. The circumference of the base is $2\pi\cdot8=16\pi$, and so the area of the rectangle is $17\cdot32\pi=544\pi$.

15. Find $(2+3\imath)-(3-4\imath)$.
Answer: –1 + 7$\imath$
$(2+3\imath)-(3-4\imath)=(2-3)+(3-(-4))\imath=-1+7\imath$.

16. In how many ways can Ally, Brett, Candy, and Doug be lined up so that Ally is standing next to her best friend Doug?
Answer: 12
Treat Ally and Doug as one unit since they must stand together. Since there are three entities, there are $3!=3\cdot2\cdot1=6$ ways to line them all up. Since Ally can be standing either to the left or to the right of Doug, there are $2\cdot6=12$ possible orderings of the four people.

17. If 9253824 beads are split among 12 people so that each person has the same whole number of beads, how many are left over?
Answer: 0
9253824 is divisible by 4 since the number made by its last two digits is divisible by 4. Also, since the sum of the

digits of 9253824 is $9+2+5+3+8+2+4 = 11+8+10+4 = 19+14 = 33$, it is divisible by 3. Since it is divisible by both 3 and 4, it is also divisible by 12, and there are no beads left over.

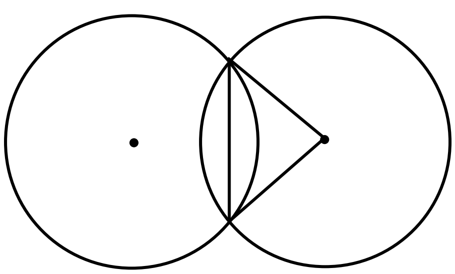
18. In how many ways can 10 be written as a sum of three (not necessarily distinct) positive integers?
Answer: 8
The least of these three integers must be no greater than 3; otherwise, the sum would be at least 12. If it is 3, the other two numbers must be 3 and 4. If it is 2, the other two numbers can be 2 and 6 or 3 and 5 or 4 and 4. If it is 1, the other two numbers can be 1 and 8 or 2 and 7 or 3 and 6 or 4 and 5. This makes $1+3+4 = 8$ possibilities.

19. A group of people are trying to sit themselves in rows. If they sit in rows of four, there are three people left over. If they sit in rows of five, there are four people left over. If they sit in rows of six, there are five people over. What is the smallest possible number of people in this group?
Answer: 59
If there were one more person, the people would be able to sit themselves in rows of 4, 5, and 6. The smallest possible number of people is thus one less than the least common multiple of 4, 5, and 6. The least common multiple of these three has two factors of 2, a factor of 3, and a factor of 5, and is thus $2^2 \cdot 3 \cdot 5 = 3 \cdot 4 \cdot 5 = 60$, and our answer is 59.

20. While on page n of her book, Alice realizes that the product of the page number before this page (page $n-1$) and the product of the page number after this page (page $n+1$) is 168. What is n?
Answer: 13
We are given $(n-1)(n+1) = 168 \Rightarrow n^2 - 1 = 168 \Rightarrow n^2 = 169 \Rightarrow n = 13$.

21. Austin runs an animal hospital that takes care of cats and dogs. The cats here only have three legs and one tail each and the dogs have six legs each and two tails. Victoria, the hospital inspector, walks in one day and counts 36 legs and 12 tails. If n is the number of animal heads in the hospital, find the sum of all possible values of n. There may be zero cats or zero dogs.
Answer: 63
Let c be the number of cats and d be the number of dogs. Then $3c+6d = 36 \Rightarrow c+2d = 12$ and $c+2d = 12$. Any ordered pair (c,d) that makes $c+2d = 12$ is a possibility for the number animals in the hospital. There are 7 possibilities, each of which give a different value of n: $(0,6)$, $(2,5)$, $(4,4)$, $(6,3)$, $(8,2)$, $(10,1)$, and $(12,0)$.

22. Two congruent circles with radii 6 have centers that are $6\sqrt{3}$ units apart. What is the area of the union of the two circles?
Answer: $60\pi + 18\sqrt{3}$
If the circles didn't overlap, the area of the union would be $2 \cdot \pi \cdot 6^2 = 72\pi$. However, in this case, this calculation counts the overlap area twice. We will find the area of half this overlap by subtracting the area of an isosceles triangle with vertex at the center of one circle and base a chord of the same circle (see drawing) from the area of the circular sector bounded by the triangle's legs.



Since this isosceles triangle is in fact an equilateral triangle (the base, which has length $2\sqrt{6^2-(3\sqrt{3})^2} = 2\sqrt{36-27} = 2\sqrt{9} = 6$, has the same length as the legs), the angle at the center of the sector is 60^o. Thus, the area of the sector is $\frac{1}{6} \cdot 36\pi = 6\pi$. The area of the equilateral triangle is $\frac{1}{2} \cdot 6 \cdot 3\sqrt{3} = 9\sqrt{3}$. Therefore, the area of the entire intersection is $2(6\pi - 9\sqrt{3}) = 12\pi - 18\sqrt{3}$, and the area of the union of the two circles is $72\pi - (12\pi - 18\sqrt{3}) = 60\pi + 18\sqrt{3}$.

23. How many perfect squares have three digits or less?

Answer: 32

Since $31^2 = 961 < 1000 < 32^2 = 1024$, there are $31 - 0 + 1 = 32$ perfect squares with three digits or less. (0 is a perfect square!!)

24. I have only quarters, dimes, nickels, and pennies in my pocket. I count 93 cents in my pocket. What is the difference between the largest number of coins I could possibly have and the smallest number of coins I could possible have? There might be none of a certain type of coin.

Answer: 85

The largest possible number of coins occurs when there are 93 pennies. The smallest possible of coins occurs when there are 3 quarters, a dime, a nickel, and 3 pennies for a total of 8 coins. The difference is $93 - 8 = 85$.

25. Find the sum of all positive composite numbers less than or equal to 20.

Answer: 132

The composite numbers less than or equal to 20 are 4, 6, 8, 9, 10, 12, 14, 15, 16, 18, and 20. Their sum is $4 + 6 + 8 + 9 + 10 + 12 + 14 + 15 + 16 + 18 + 20 = 10 + 17 + 22 + 29 + 34 + 20 = 27 + 51 + 54 = 132$.

26. Convert 3018, a number in base 10, to base 16. Give your answer without the subscript 16.

Answer: BCA

Since $16^2 = 256 < 3018 < 4096 = 16^3$, we will only need three digits. $\left\lfloor \dfrac{3018}{256} \right\rfloor = 11$, so the first digit is B. Next, $3018 - 11 \cdot 256 = 202$. $\left\lfloor \dfrac{202}{16} \right\rfloor = 12$, so the second digit is C. Finally, since $202 - 12 \cdot 16 = 10$, the third digit is A and $3018_{10} = BCA_{16}$.

27. Let $x^{\underline{n}} = x(x-1)(x-2)\ldots(x-n+1)$. Find $8^{\underline{3}}$.

Answer: 336

$8^{\underline{3}} = 8 \cdot 7 \cdot 6 = 56 \cdot 6 = 336$.

28. Tom makes half of his 3-point shots. What is the probability that Tom makes at least five out of nine 3-point shots?

Answer: $\frac{1}{2}$

The probability of Tom making at least five of nine shots is the same as him making four or less shots. (This doesn't work if the probability that Tom makes any shot is not $\frac{1}{2}$). Our answer is $\frac{1}{2}$.

29. Compute the ratio of the area of a square to that of its inscribed circle.

Answer: $\frac{4}{\pi}$

Let the side length of the square be s. Then the area of the square is s^2, the radius of the circle is $\frac{s}{2}$, and the area of the circle is $\dfrac{\pi s^2}{4}$. Thus our ratio is $\dfrac{s^2}{\frac{\pi s^2}{4}} = \dfrac{4}{\pi}$.

30. If $3a - 2b = 11$ and $5a + 7b = 163$, find $(a+b)^2$.

Answer: 729

Add seven times the first equation to twice the second to obtain $31a = 403 \Rightarrow a = 13$. Plugging in to the first equation, $3 \cdot 13 - 2b = 11 \Rightarrow -2b = -28 \Rightarrow b = 14$, and $(13 + 14)^2 = 27^2 = 729$.

31. If a, b, and c are positive real numbers such that $(a-b)^2 + (b-c)^2 + (c-a)^2 = 0$ and $a + b + c = 9$, then find the product abc.

Answer: 27

$x^2 \geq 0$ for all real x. Thus, by the first equation, we know $a = b = c$. By the second equation, $a = b = c = 3$, and $abc = 27$.

32. If $f(x) = \dfrac{1}{10x-5}$ for $1 \le x \le 10$, then find the maximum value of $f(x)$ on this range minus the minimum value of $f(x)$ on this range.
Answer: $\frac{18}{95}$
The maximum value comes when $x = 1$ since that minimizes the denominator. This maximum value is $\frac{1}{5}$. The minimum value comes when $x = 10$ since that maximizes the denominator. This minimum value is $\frac{1}{95}$. The difference is $\dfrac{1}{5} - \dfrac{1}{95} = \dfrac{19-1}{95} = \dfrac{18}{95}$.

33. Evaluate $\sum_{n=1}^{10} n^2$.
Answer: 385
The sum is $1^2+2^2+3^2+\ldots+10^2 = 1+4+9+16+25+36+49+64+81+100 = 5+25+61+113+181 = 40+174+181 = 385$. (The sum of the first n squares is $\sum_{k=1}^{n} k^2 = \dfrac{n(n+1)(2n+1)}{6}$.)

34. A cubic ice cube weighs 920 grams. If the density of ice is .92 g/cm^3, then at least how many such ice cubes would it take to build a tower that is three meters tall? There are 100 cm in a meter.
Answer: 30
The ice cube has a volume of $\dfrac{920}{.92} = 1000$ cm^3, which means it must have side length 10 cm. Since 3 meters is the same as 300 centimeters, the minimum number of cubes needed is $\dfrac{300}{10} = 30$.

35. Sam and Sherry are playing a game of Brawl using their favorite characters, Pit and Olimar, respectively. Sam's percentage probability of winning is 55% while Sherry's percentage probability of winning is 45%. After two one-on-one matches, what is the probability that each has won a match?
Answer: $\frac{99}{200}$
The probability that each wins a match is $2 \cdot \dfrac{55}{100} \cdot \dfrac{45}{100} = 2 \cdot \dfrac{11}{20} \cdot \dfrac{9}{20} = 2 \cdot \dfrac{99}{400} = \dfrac{99}{200}$. (There are two orderings of the matches; either Sherry wins the first match or the second.)

36. Two positive integers are relatively prime if they share no factors other than 1. $\phi(n)$ is the defined as the number of numbers less than or equal to n that are relatively prime to n. For example, $\phi(6) = 2$. Find $\phi(35)$.
Answer: 24
Any number that is not divisible by 5 or 7 is relatively prime to 35. There are $\dfrac{35}{5} = 7$ multiples of 5 less than or equal to 35, and $\dfrac{35}{7} = 5$ multiples of 7 less than or equal to 35. Finally, $\phi(35) = 35 - 5 - 7 + 1 = 24$ (we counted 35 twice since it is a multiple of 5 and 7).

37. How many integer solutions (a, b), with $a < b$, does the equation $ab + a + b = 17$ have?
Answer: 3
Add 1 to both sides to obtain $ab+a+b+1 = 18 \Rightarrow (a+1)(b+1) = 18$. Each factor pair of 18 corresponds to a solution (a, b) to the equation (a factor pair of a number n is a pair of integers p and q with $pq = n$). There are three factor pairs of 18: (1,18), (2,9), and (3,6), so our answer is 3.

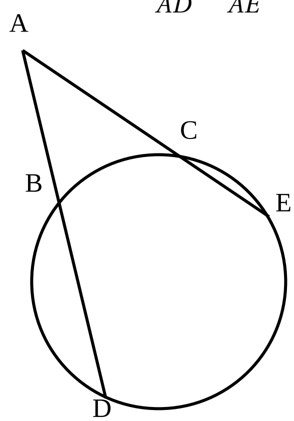
38. $n! = n(n-1)(n-2)\ldots 3 \cdot 2 \cdot 1$. Find the greatest integer a such that 50! is divisible by 2^a.
Answer: 47
We must count the number of factors of 2 in $50 \cdot 49 \cdot 48 \cdot \ldots \cdot 3 \cdot 2 \cdot 1$. Each even number contributes one factor of 2, for a total of 25. Each multiple of 4 contributes another multiple of 2, for a total of 12 more. Each number divisible by 8 contributes another factor of 2, which makes 6 more. Similarly, multiples of 16 and 32 contribute 3 and 1 more factor of 2, respectively, and the greatest a that makes the statement true is $25 + 12 + 6 + 3 + 1 = 47$.

39. Find the ratio of volume to surface area of a cube with side length 18.
Answer: 3

The volume of a cube with side length s is s^3, and the surface area of a cube with side length s is $6s^2$. The ratio of volume to surface area is $\dfrac{s^3}{6s^2} = \dfrac{s}{6}$. For this particular cube, that ratio is $\dfrac{18}{6} = 3$.

40. What is the length of the median to the hypotenuse in a $6-8-10$ right triangle?
Answer: 5
Consider the circumscribed circle of any right triangle. The central angle of the arc subtended by the 90^o angle is $2\cdot 90^o = 180^o$, so the hypotenuse of any right triangle is the diameter of the triangle's circumcircle, and the midpoint of the hypotenuse is the center of the circle. Thus, the length of the median to the hypotenuse in a right triangle is half the hypotenuse, or $\frac{10}{2} = 5$.

41. In the drawing, $\overline{AB} = 7$, $\overline{AD} = 16$, and $\overline{AC} = 8$. Compute $\overline{CE}$.
Answer: 6
Draw $\overline{CD}$ and $\overline{BE}$. $\triangle ACD \sim \triangle ABE$ since $\angle A = \angle A$ and $\angle BDC = \angle BEC$ since they subtend the same arc. From the similarity, we know $\dfrac{\overline{AC}}{\overline{AD}} = \dfrac{\overline{AB}}{\overline{AE}} \Rightarrow \overline{AC}\cdot\overline{AE} = \overline{AB}\cdot\overline{AD}$. Thus, $\overline{AE} = \dfrac{7\cdot 16}{8} = 7\cdot 2 = 14$ and $\overline{CE} = 14 - 8 = 6$.



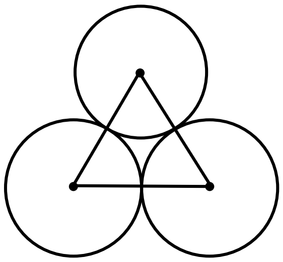
42. In $\triangle ABC$, $\overline{AB} = 9$ and $\overline{BC} = 8$. If $\angle ABC = 30^o$, then what is the area of $\triangle ABC$?
Answer: 18
Draw the altitude to C, and let it intersect $\overline{AB}$ at D. Then $\triangle DBC$ is a 30-60-90 right triangle and $\overline{CD} = \dfrac{1}{2}\cdot\overline{BC} = 4$. Finally, the area of the triangle is $\frac{1}{2}\cdot 9\cdot 4 = 18$.

43. Compute $\sqrt{2+\sqrt{2+\sqrt{2+\ldots}}}$.
Answer: 2
Set $a = \sqrt{2+\sqrt{2+\sqrt{2+\ldots}}}$. Then $a^2 - 2 = a \Rightarrow a^2 - a - 2 = 0 \Rightarrow (a-2)(a+1) = 0 \Rightarrow a = 2$ since a clearly cannot be negative.

44. $\lfloor x \rfloor$ is defined as the greatest integer less than or equal to x. For example, $\lfloor 5.5 \rfloor = 5$, $\lfloor \pi \rfloor = 3$, and $\lfloor 4 \rfloor = 4$. Compute the values of x for which $\lfloor x\lfloor x \rfloor + x \rfloor = 2$.
Answer: -2 and 1≤x<1.5
We must have $2 \le x\lfloor x \rfloor + x < 3$. If $2 \le x < 3$, this inequality is $2 \le 3x < 3 \Rightarrow \frac{2}{3} \le x < 1$, a contradiction. If $1 \le x < 2$, the inequality is $2 \le 2x < 3 \Rightarrow 1 \le x < 1.5$. If $-2 \le x < -1$, then the inequality is $2 \le -x < 3 \Rightarrow -3 < x \le -2 \Rightarrow x = -2$. It is left to the reader to show there are no more solutions.

45. Each of three congruent circles is externally tangent to the other two. If the radius of each circle is 4, then find the area of the region in the middle of the three circles.
Answer: $16\sqrt{3} - 8\pi$
Connect the three centers of the circles to create an equilateral triangle. The area we want is the area of the triangle minus three 60^o degree sectors, which are the same as half the area of a circle. Since the area of an equilateral

triangle with side length s is $\frac{s^2\sqrt{3}}{4}$, the area of this equilateral triangle is $16\sqrt{3}$. Also, the area of half a circle is $\frac{1}{2} \cdot 16\pi = 8\pi$, and our answer is $16\sqrt{3} - 8\pi$.



46. The real numbers $a_1, a_2, a_3, \ldots, a_{20}$ are written in that order around a circle. Given that $a_1 = 1$, $a_11 = 2$, $a_14 = 3$, and that the sum of any four consecutive terms is 20, find a_4.

Answer: 14

Since the sum of any four consecutive terms is 20, we have $a_1 + a_2 + a_3 + a_4 = 20 = a_2 + a_3 + a_4 + a_5 \Rightarrow a_1 = a_5$. Similarly, $a_5 = a_9$. Continuing this process, we see that $a_m = a_n$ if and only if $m \equiv n \pmod 4$. So $a_1 = a_1$, $a_{14} = a_2$, $a_{11} = a_3$, and $a_4 = a_4$. The sum of all twenty of the numbers is $\frac{(a_1 + a_2 + a_3 + a_4) + (a_2 + a_3 + a_4 + a_5) + \ldots + (a_{20} + a_1 + a_2 + a_3)}{4}$ since each term is counted in four expressions. This fraction is equal to $\frac{20 \cdot 20}{4} = 100$. The sum of all the terms is also equal to $(a_1 + a_5 + \ldots + a_{17}) + (a_2 + a_6 + \ldots a_{18}) + (a_3 + a_7 + \ldots + a_{19}) + (a_4 + a_8 + \ldots + a_{20}) = 5a_1 + 5a_2 + 5a_3 + 5a_4 = 5(a_1 + a_2 + a_3 + a_4) = 100 \Rightarrow a_1 + a_2 + a_3 + a_4 = 20 \Rightarrow 1 + 2 + 3 + a_4 = 20 \Rightarrow a_4 = 14$.

47. If $x^3 + ax^2 - 6x - 12$ is evenly divisible by $x - 1$, then find a.

Answer: 17

If the cubic is evenly divisible by $x - 1$, then 1 is a solution to the equation $x^3 + ax^2 - 6x - 12 = 0 \Rightarrow 1 + a - 6 - 12 = 0 \Rightarrow a - 17 = 0 \Rightarrow a = 17$.

48. Compute $\log_{10}\left(\frac{1}{2}\right) + \log_{10}\left(\frac{2}{3}\right) + \log_{10}\left(\frac{3}{4}\right) + \ldots + \log_{10}\left(\frac{999}{1000}\right)$.

Answer: –3

Since $\log a + \log b = \log ab$, we have $\log_{10}\left(\frac{1}{2}\right) + \log_{10}\left(\frac{2}{3}\right) + \log_{10}\left(\frac{3}{4}\right) + \ldots + \log_{10}\left(\frac{999}{1000}\right) = \log_{10}\left(\frac{1}{2} \cdot \frac{2}{3} \cdot \frac{3}{4} \cdot \ldots \cdot \frac{999}{1000}\right) = \log_{10}\left(\frac{1}{1000}\right) = -3$ since $10^{-3} = \frac{1}{1000}$.

49. If there are 6,000,000,001 people in the world and each has less than 100,000 hairs on his or her head, find the maximum number of people that MUST have the same number of hairs on their head.

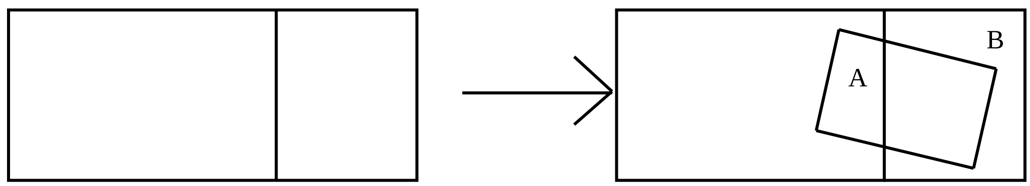
Answer: 60,001

In the world in the problem, there are 100,000 possibilities for the number of hairs on a person's head: 0, 1, 2, … 99,999. By the Pidgeonhole Principle, this means that at least $\left\lceil \frac{6,000,000,001}{100,000} \right\rceil = 60,001$ people must have the same number of hairs on their heads.

50. Square $ABCD$ has side length 4. Points E and F are on sides $\overline{AB}$ and $\overline{BC}$, respectively, such that $\frac{\overline{AE}}{\overline{EB}} = 2$ and $\frac{\overline{BF}}{\overline{FC}} = 3$. Find the difference between the area inside the square but outside both $\triangle CDE$ and $\triangle ADF$ and the area inside both $\triangle CDE$ and $\triangle ADF$.

Answer: 0

Consider two regions that cover an entire third region and do not overlap. It is easy to see that if one (or both) of the first two regions is moved, then the area of the overlap of the two regions is equal to the area inside the third region but outside the first two. Since the area of each of $\triangle CDE$ and $\triangle ADF$ is 2 (and thus the sum of their areas is the area of the square), the difference asked for in the question must be 0.



(The area of region A equals that of region B.)