7th Annual Bergen County Academies Math Competition

Fifth Grade

Sunday, 18 October 2009

1 Rules

1. You may use space on your test paper and additional scrap paper to do work. Your answers must be written on the answer sheet. We will not look at answers written on your test paper.
2. Each problem has only one answer. If you put more than one answer for a problem, you will be marked wrong. When changing an answer, be sure to erase or cross out completely.
3. Write legibly. If the graders cannot read your answer, it will be marked incorrect.
4. Fractions should be written in lowest terms. For example, if the answer is $\frac{1}{2}$, then $\frac{2}{4}$ will not be accepted although the two fractions are numerically equal.
5. All other answers should be written in simplest form.
6. If a unit is indicated in the problem, the answer must be given in that unit. For instance, if the problem asks for the answer in hours, you cannot give your answer in minutes. Furthermore, you don't need to write the unit, as the graders will assume your answer is in the units asked for in the problem.
7. There is no penalty for guessing.
8. Ties will be broken based on the number of correct responses to the last ten questions. If a tie remains, then the correct responses to the last five questions will break the tie.
9. We will announce how much time is remaining often during the test.

2 Contest

1. If $100 \cdot (1.9 + 9.1) = 10 \cdot (28 + x)$, what is x?
2. Josh is thinking of two numbers from the set $\{1, 2, 3, 4, 5, 6, 7, 8\}$. Their sum is 9. How many possible values are there for the smaller of the two numbers?
3. $(2 \cdot 2009) + (4 \cdot 2009) + (6 \cdot 2009) = ? \cdot 2009$
4. Complete the following sequence: 1, 2, 4, 4, 9, 8, 16, 16, 25, 32, ?
5. Evaluate $1 - 2 - 3 + 4 + 5 - 6 - 7 + 8 + 9 - ... + 2009$.
6. Jenny is rearranging the salt and pepper shakers in a restaurant. There are four salt shakers and five pepper shakers. All the salt shakers are indistinguishable and all the pepper shakers are indistinguishable. How many ways can she arrange them in a line such that the salt and pepper shakers alternate?
7. Find the value of x given the following equations:

$$3 \cdot x - 4 \cdot y = 2$$

$$5 \cdot y = 2 - 4$$

8. On Steve's farm, there are chickens (which have one head and two legs), and headdie-horses (which have three heads and five legs). Richard decided to count the heads and legs on the farm. He counted forty-three heads and seventy-nine legs. How many chickens were there?
9. Find the twentieth positive odd number.
10. A rectangular prism has two faces with area 6, two faces with area 18, and two faces with area 300. Find its volume.
11. A car and truck are traveling toward each other. The car travels at 20 miles per hour and the truck travels at 30 miles per hour. If they were originally 150 miles apart, after how many hours do they meet?
12. How many factors does 1001 have?
13. Alex Zhu was asked to find the sum of two nonnegative integers. Instead, he found their product. Luckily, his answer was the same. Find all possible values of the sum of those two integers.
14. Simplify the following expression: $\frac{60}{32} \cdot \frac{12}{14} \cdot \frac{28}{45}$.
15. Sherry's rectangular garden measures 5 feet wide by 4 feet long. If she wants to plant potatoes on the perimeter of the garden such that each potato is at least a foot away from any other potato, what is the maximum number of potatoes she can plant?
16. What is the sum of the first 20 positive multiples of 3?
17. If Gary drives to Peter's house at 50 miles per hour, it will take him 2 hours less than if he bikes to Peter's house at 10 miles per hour. How far away does Gary live from Peter's house, in miles?
18. Find the number of integer solutions to the equation $x^2 + y^2 = 103$.
19. Julia is playing darts. On each turn, she can score 5 or 7 points. What is the maximum number of points she cannot get?

20. Find the number of subsets S, including the empty set, of $\{1, 2, 3, ..., 10\}$ such that if $x \neq 10$ and x is in S, then $x+1$ is also in S.

21. Alex Zhu has suggested that the BCA math competition have 50 normal problems worth 1 point each, 5 hard problems worth 2 points each, and 10 very hard problems worth 3 points each. If this were so, find the maximum possible score that a student could get.

22. If bagels are only sold in bags of 17, what is the least number of bags I must buy to get 103 bagels?

23. What is 9950.0599 rounded to the nearest hundred?

24. Ben buys a sharpie and two rulers. The total cost is \$2.25. If a sharpie costs \$0.50 more than a ruler, how much would it cost to buy four sharpies and three rulers?

25. How many integers n are there such that $\frac{120}{n}$ is also an integer?

26. Let $\lfloor x \rfloor$ denote the greatest integer less than or equal to x. Given that $\pi \approx 3.14159265...$, compute $\lfloor 203\pi \rfloor$.

27. Alice flips a coin four times. What is the probability that she will get exactly one head?

28. Julia has \$10. Her grandmother returns home from China and gives her \$200. Find the percent increase in Julia's money.

29. Define $x\%y = \dfrac{2 \cdot x + y}{x - y}$.What is $(5 \cdot (5\%2))\%10$?

30. Mark and Dan are racing against each other. Dan can run at 1 mile per hour and Mark can run at 15 miles per hour. Dan starts the race 70 miles ahead of Mark. How many hours does it take for Mark to catch up to Dan?

31. Find the missing number: $\dfrac{55 \cdot 77}{?} = 5 \cdot 7$

32. A piece of plastic food wrap measures 12 inches by 3000 feet. What is the area in square feet?

33. Find the two points where at least two of lines $y = x + 1$, $x + y = 1$, and $x = y + 1$ intersect.

34. Matt, Robert, and Nikhil are dividing a pile of chocolates. Matt takes half of the pile and then takes three more chocolates. Robert then takes half of the pile and then takes three more chocolates. Nikhil then takes half of the pile and then takes three more chocolates. After that, Jordan passes by to collect the last remaining chocolate. How many chocolates were there in the initial pile?

35. What is the product of the all the integers from -10 to 10 inclusive?

36. Paul eats 2100 calories of food per day. He wants no more than 15% of his calories to come from fat. If each gram of fat provides nine calories, what is the maximum number of grams of fat that Paul can eat in one day?

37. What is $2\dfrac{2}{3}$ subtracted from its reciprocal?

38. A man was born in a perfect cube year in the 18th century and died in a perfect square year in the same century. For how many years did the man live?

39. Kevin Koh is preparing a mixture of potassium hydroxide. If he has 1 liter of a solution of evenly mixed potassium hydroxide that has a total of 2 grams potassium hydroxide, find the amount of that solution he must add into another solution with 0.6 liters of water and no potassium hydroxide to get a final solution that has $\frac{4}{3}$ grams of potassium hydroxide?

40. How many integers from 1 to 100 are multiples of 2 and 3 but not 5?

41. Patricia is creating a daily schedule. She notices that she goes to school for one-third of the day, does homework for 12.5% of the day, plays tennis for three hours, and sleeps for the rest. Assuming she never does two things at once, what fraction of the day does Patricia spend sleeping?

42. Alex is preparing for the BCA Math Competition. On the nth day, he does $2 \cdot n - 1$ problems. How many problems will Alex have done after the tenth day?

43. Sue has a set of ten numbers. How many ways can she pick eight of them?

44. A circle with center O has points A and B on its circumference. The area of the circle is 36π and $\angle AOB = 90^o$. Compute $\overline{AB}$.

45. Yi is playing a game. He starts at the point $(1, 0)$. First he must run to the y-axis, then he must run to the point $(3, 4)$. What is the least distance that he has to run?

46. Kevin drives from his home to his local library at thirty miles per hour. He makes the return trip at twenty miles per hour. In miles per hour, what is his average speed during the trip from his home to the library and back?

47. If three six-sided dice are rolled, what is the probability that their sum is 17?

48. There are lights A,B,C,D,E,F,G in this order in a line. Every light has a switch. Right now lights A,C,E,G are on and the rest are off. Starting at A and walking to G, Alex turns the switch of each light once. He repeats this process again and again from A to G. After exactly 1999 switches have been turned, which lights are on?

49. If $x^2 + 1 = 2 \cdot x$, then what is x?

50. Jim sells 140 tickets for \$2001, some at full price and others half-price. Tickets sell for a whole number of dollars. How much money is raised by half-price tickets?