7th Annual Bergen County Academies Math Competition

Fourth Grade

Sunday, 18 October 2009

1 Contest

1. Compute $9 \cdot (1.0000 + 0.10000 + 0.01000 + 0.00100 + 0.00010 + 0.00001) + .00001$.

   Solution: This is equal to $9 \cdot (1.11111) + .00001 = 9.99999 + .00001 = \boxed{10}$.

2. At a tennis tournament, every player plays exactly one match against each of the other players. How many matches will be played if twenty people participate?

   Solution: Since each player plays each of the other players once, then each player will play 19 matches. Thus, there will be $(20)(19) = 380$ matches. However, we have counted each match twice since we counted both participants, so our final answer is $\frac{380}{2} = \boxed{190}$.

3. If $x + 2 \times x + 3 \times x = 6$, find x.

   Solution: $x + 2x + 3x = (1 + 2 + 3)x = 6x$, so we must solve $6x = 6$. Dividing both sides by 6 yields $x = \boxed{1}$.

4. In how many ways can 7 be written as a sum of three (not necessarily distinct) positive whole numbers? The order of the numbers does not matter.

   Solution: $7 = 1 + 1 + 5 = 1 + 2 + 4 = 1 + 3 + 3 = 2 + 2 + 3$. Any other sum will just be re-ordering of one of these, so there are $\boxed{4}$ ways to write 7 as a sum of three positive whole numbers.

5. If the ratio of the circumference to the area of a circle is $1 : 3$, what is its radius?

   Solution: Let r be the radius of the circle. We know that the circumference is $2\pi r$ and the area is πr^2, so we must solve $\frac{2\pi r}{\pi r^2} = \frac{1}{3}$, or $\frac{2}{r} = \frac{1}{3}$. Cross-multiplying gives us our answer of $r = \boxed{6}$.

6. Sherry and Jenny were going for a walk when they noticed that the rocks on the pathway formed a pattern. The first stone was light gray, the second stone was medium gray, the third stone was dark gray, the fourth stone was white, the fifth stone was light gray, etc. If the pattern continues, what color will the 333rd stone be?

   Solution: The pattern repeats after every four stones. Since the first stone was light gray, the $(4x+1)$th stone is light gray, for all positive integers x. Since $333 = 332 + 1 = 4 \times 83 + 1$, which is in the form $4x + 1$, the 333rd stone must be $\boxed{\text{light gray}}$.

7. In a zoo with thirty animals: some are rabbits and the rest are penguins, Hannah counts thirty-five pairs of legs. How many penguins are there?

   Solution: There is one pair of legs per penguin and 2 pairs of legs per rabbit. Let p be the number of penguins, and let r be the number of rabbits. So $p + r = 30$ and $p + 2r = 35$. If we subtract the first equation fromm the second, we get

$$p + 2r - (p + r) = 35 - 30$$
$$r = 5$$

Plugging that into the equation $p + r = 30$, we find that $p = 25$, and there are $\boxed{25}$ penguins.

8. A hamburger costs \$5.95 and a soda costs \$2.55. In cents, how much does Kun-soo pay to buy two hamburgers and two sodas?

   Solution: It is clear that a hamburger costs 595 cents and a soda costs 255 cents. The problem requires us to compute $(2)(595) + (2)(255) = 1190 + 510 = \boxed{1700}$ cents.

9. If $a + b + c = 5$ and $2 \times a + 2 \times b + c = 8$, what is the value of c?

   Solution: Subtracting the first equation $(a + b + c = 5)$ from the second equation $(2a + 2b + c = 8)$ results in $a + b = 3$. Looking at the first equation again, we have

$$\begin{aligned} a + b + c &= 5 \\ (a + b) + c &= 5 \\ 3 + c &= 5 \end{aligned}$$

   which gives us the desired answer of $c = \boxed{2}$.

10. The Bergen County Academies math test has fifty problems and lasts one hour and thirty minutes. If Bob decides to spend the same amount of time on each problem, how many minutes will he spend on problem 42?

    Solution: With 90 minutes (one hour is 60 minutes, so an hour and thirty minutes is $60 + 30 = 90$ minutes) and fifty problems he must spend $\frac{90}{50} = \frac{9}{5}$ minutes per problem. Hence he spends $\boxed{\frac{9}{5} = 1\frac{4}{5} = 1.8 \text{ minutes}}$ on problem 42.

11. Evaluate $12 + 34 \times 56$.

    Solution: This can all be done by hand: $34 \cdot 56$ is 1904. Add 12 to get $\boxed{1916}$.

12. David and Zuming are dividing thirty apples. David only wants a multiple of two apples while Zuming only wants a multiple of three apples. Each wants at least one apple. How many ways can they divide the apples amongst themselves so that no apples are left over?

    Solution: Let x and y be positive integers, let $2x$ be the number of apples that David gets, and let $3y$ be the number of apples that Zuming gets. Then

$$\begin{aligned} 2x + 3y &= 30 \\ 2x - 2x + 3y &= 30 - 2x \\ 3y &= 30 - 2x \\ \frac{3y}{3} &= \frac{30 - 2x}{3} \\ y &= \frac{30}{3} - \frac{2x}{3}, \end{aligned}$$

    so $y = 10 - \frac{2x}{3}$. So y will only be a positive integer if $\frac{2x}{3}$ is also one and is less than 10. So if $n = \frac{x}{3}$ is a positive integer, then $y = 10 - 2n$. Since $2n$ must be greater than 0 and less than 10, n can be 1, 2, 3, or 4. Thus, there are also 4 choices for (x, y), and $\boxed{4 \text{ ways}}$ to split the apples.

13. What is the largest integer n such that n and n+13 are both perfect squares?

    Solution: Let $n = a^2$ and let $n + 13 = b^2$, where a and b are both whole numbers. Subtracting these equations yields $13 = b^2 - a^2 = (b - a)(b + a)$. Since $b - a$ and $b + a$ are both positive integers which are factors of 13, they must be equal to 1 and 13, respectively, since 13 is prime. Thus, since $b - a = 1$ and $b + a = 13$, subtracting the first equation from the second yields $2a = 12$, or $a = 6$, and $n = a^2 = \boxed{36}$.

14. Kevin is stacking blocks in a pyramid. The first layer contains two blocks, the second layer contains four blocks, the third layer contains six blocks, and so on. If the pyramid has seventeen layers, how many blocks is it composed of?

    Solution: The number of blocks is simply $2+4+6+8+10+12+14+16+18+20+22+24+26+28+30+32+34$, which simplifies to $\boxed{306}$.

    OR

    Solution: Again, the number of blocks is $2+4+6+8+10+12+14+16+18+20+22+24+26+28+30+32+34$, but instead of doing the entire sum by hand, we can reorder the terms like this: $(2+34)+(4+32)+(6+30)+(8+28)+(10+26)+(12+24)+(14+22)+(16+20)+18$. This equals $36+36+36+36+36+36+36+36+18 = 8\times 36+18 = 288+18 = \boxed{306}$.

15. What is $2+2\times(2+2\times(2+2\times(2+2\times 2)))$?

    Solution: This is equivalent to $2+2\times(2+2\times(2+2\times 6)) = 2+2\times(2+2\times 14) = 2+2\times 30 = \boxed{62}$.

16. Many integers can be expressed as a sum of 3 perfect squares. For example, $5 = 2^2+1^2+0^2$. 7 is the smallest positive integer which cannot be expressed in this way. What is the next smallest integer that cannot be expressed as a sum of 3 perfect squares?

    Solution: From brute force, the answer is simply $\boxed{15}$.

17. Anand bought a 72-foot long wire fence for his rectangular garden and wants to surround the biggest possible area. One side of the garden is adjacent to his house, so he only needs to surround three sides. In square feet, what is the biggest area he can surround?

    Solution: Let a be the length of the sides of the fence that are perpendicular to the house, and let b be the length of the side of the fence that is parallel to the house. Then $2\times a+b = 72$, and we wish to maximize area $= a\times b$. This happens when the fence forms a square, so $a = b$. We then have $2\times a+b = 2\times a+a = 3\times a = 72$, so $a = b = 24$. Then the maximum area is $24\times 24 = \boxed{576 \text{ square feet}}$.

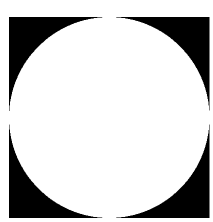
18. Define a?b to be equal to $\dfrac{2\times a+3\times b}{a-b}$. Find (3?4)?2.

    Solution: (3?4)?2 is equivalent to $\left(\dfrac{2\times 3+3\times 4}{3-4}\right)?2 = (-18)?2 = \dfrac{2\times -18+3\times 2}{-18-2} = \dfrac{30}{20} = \boxed{\dfrac{3}{2} = 1\dfrac{1}{2} = 1.5}$.

19. If Starbursts are only sold in bags of 6, what is the least number of bags Austin must buy to get 49 Starbursts?

    Solution: Buying 8 bags will get $8\times 6 = 48$ Starbursts, which is not enough. Buying 9 bags will get $9\times 6 = 54$ Starbursts, which is more than enough. Therefore, the least number needed is $\boxed{9 \text{ bags}}$.

20. A circle is inscribed in a square with a side length of 2. What is the area of the square not inside the circle?



Solution: Since the circle is inscribed in the square, its radius is half the side length of the square. Since the side length is 2, the radius r of the circle is 1. The desired area can be found by subtracting the area of the circle, $\pi \times r^2 = \pi$, from the area of the square, $2 \times 2 = 4$, which is $\boxed{4-\pi}$.

21. How many positive integers less than 80 have exactly four factors?

    Solution: An integer n with two distinct prime factors p and q, and $n = p \times q$, will have exactly four factors (1, p, q, and $p \times q$). The possible values of p and q are 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, and 37 (41, the next largest prime, is too big, because $2 \times 41 > 80$). Since order doesn't matter, we can make the assumption that $p > q$. Then, if p is 37, 31, or 29, $q = 2$ (q can't be 3 or higher because then $p \times q$ would be greater than 80); this accounts for three possible integers. If p is 17, 19, 23, then q is either 2 or 3, which gives 6 possible combinations. If p is 13, then q is 1, 3, or 5 (3 more combinations); if p is 11, q can be 2, 3, 5, or 7 (4 combinations). $p = 7$ gives 3 combinations (q can be 2, 3, or 5), $p = 5$ gives two combinations, and $p = 3$ gives one combination. Adding up these different cases, we find that the total number of possibilities is $3+6+3+4+3+2+1 = \boxed{22}$.

22. The angles A, B, C, and D ($A < B < C < D$) in a quadrilateral form an arithmetic series. What is $\dfrac{B+C}{A+D}$?

    Solution: Since the angles form an arithmetic sequence, then B, C, and D can be expressed in terms of A, namely $A+d$, $A+2d$, and $A+3d$, respectively, for some real number d. Plugging the expressions for B, C, and D into the fraction, one gets $\frac{A+n+A+2n}{A+A+3n} = \frac{2A+3n}{2A+3n}$, which simplifies to $\boxed{1}$.

23. Alex Zhu goes to school for nine hours. When he gets home at 5:00 P.M., he does two hours of math. He then spends an hour eating, and does math until 1:00 A.M. He then goes to sleep for seven hours. If Alex does not multitask and can go to and from school in no time, what percent of the day does he spend doing math?

    Solution: Since Alex gets home at 5:00 P.M. and does 2 hours of math before eating, he start eating at 7:00 P.M. After his hour of eating (at 8:00 P.M.), he starts doing math again until 1:00 A.M. The time between 1:00 A.M. and 8:00 P.M. is 5 hours; adding this to the 2 hours immediately after school, we see that Alex does 7 hours of math per day. There are 24 hours in a day, so he spends $\boxed{\dfrac{7}{24}}$ of the day doing math.

24. Evaluate

$$\frac{1}{\frac{1}{2}+\frac{1}{4}}$$

    Solution: $\frac{1}{2}+\frac{1}{4} = \frac{2}{4}+\frac{1}{4} = \frac{2+1}{4} = \frac{3}{4}$. Then we have

$$\frac{1}{\frac{1}{2}+\frac{1}{4}} = \frac{1}{\left(\frac{3}{4}\right)} = 1 \times \frac{4}{3} = \boxed{\frac{4}{3} = 1\frac{1}{3}}.$$

25. Hannah has only dimes and quarters in her coin purse. She notices that if she adds five pennies, eight nickels, and four dimes to her purse, she doubles the amount of money. What is the maximum number of coins with which she could have started out?

    Solution: Let d be the number of dimes Hannah starts with, and let q be the number of quarters she starts with. Then the amount of money she originally has is, in cents, $10d + 25q$. After adding more money, she has $10 \times (d+4) + 25q + 8 \times 5 + 5 \times 1 = 10d + 40 + 25q + 40 + 5 = 10d + 25q + 85$. Since this is twice the amount we started with, we have $2 \times (10d + 25q) = 10d + 25q = 85$, or $10d + 25q = 85$. q must equal either 1 or 3; if $q = 0$, then $10d + 25q = 10d$ is a multiple of 10 and therefore can't equal 85; if $q > 3$, then $10d + 25q \geq 100 > 85$. Since 2 quarters is equivalent to 5 dimes, we can maximize the number of coins by minimizing the number of quarts; therefore, we choose $q = 1$. This gives $d = 6$, and a total of $\boxed{\text{7 coins}}$.

26. How many integers from 1 to 100 inclusive are multiples of 12 but not 15?

Solution: $1 \times 12 = 12$ is the smallest number that satisfies the above; $8 \times 12 = 96$ is the largest, so there are 8 numbers that are multiples of 12 that are between 1 and 100. Numbers that are multiples of both 12 and 15 are multiples of $\text{lcd}(12, 15) = 60$, so there is one such multiple of 12 that does not satisfy the conditions. Therefore, there are $8 - 1 = \boxed{7}$ such integers.

27. Janet has an unknown number of gumballs. If she gives one-third of her gumballs to Jim, then Janet and Jim will have the same number of gumballs. If Jim originally had 17 gumballs, how many gumballs did Janet originally have?

Solution: Let the original number of gumballs that Janet have be $3n$, where n is a positive integer. Then $2n = n + 17$, so $n = 17$. Therefore, Janet started with $3n = 3 \times 17 = \boxed{51 \text{ gumballs}}$.

28. If $x^2y = 30$, find x when $y = 120$ and $x < 0$.

Solution: Dividing both sides of the equation by y, we get

$$\frac{x^2y}{y} = \frac{30}{y}$$
$$x^2 = \frac{30}{120} = \frac{1}{4}$$

Taking a square root results in $x = +\frac{1}{2}$ or $x = -\frac{1}{2}$. Since x must be negative, then x must equal $\boxed{-\frac{1}{2}}$.

29. The sum of three consecutive even numbers is 30. What is the product of the first two numbers?

Solution: Let x be the second of the three even numbers. Then the sum of the first three even numbers is $(x - 2) + x + (x + 2) = 30$. This equation simplifies to $3x = 30$. Solving for x gets $x = 10$. The product of the first two numbers is $(x - 2) \cdot x$, and when 10 is substituted for x, the product is $8 \cdot 10 = \boxed{80}$.

30. Evaluate $(-3) \cdot (-2) \cdot (-1) \cdot \frac{1}{36} \cdot (1) \cdot (2) \cdot (3)$.

Solution: This is equal to $(-6) \cdot \frac{1}{36} \cdot 6 = -\frac{36}{36} = \boxed{-1}$.

31. Kelvin is preparing for the Intermediate Math Open. Each day, he does two more problems than he did the previous day. If he did one problem the first day, how many problems will he have done after the eleventh day?

Solution: Kelvin does 1 problem on the first day, 3 on the second day, 5 on the third day, etc. So the total number of problems Kelvin did in eleven days is $1+3+5+7+9+11+13+15+17+19+21 = \boxed{121}$.

32. What is the largest integer we cannot obtain by adding 3's and 7's?

Solution: By trial and error, we can see that it is impossible to obtain 11 as the sum of 3's and 7's. 12=3+3+3+3, 13=7+3+3, 14=7+7. For any number greater than 14, we can just add 3's to 12, 13, or 14 until we get the number we want. Therefore, all numbers greater than 11 can be obtained by adding 3's and 7's, so $\boxed{11}$ is the answer.

33. The cost of supplying food for Tim's party was \$1000. If the cost of entrance to the party is \$5, how many people need to come for Tim to have \$500 left (after paying for the food) to buy presents for himself?

Solution: Tim needs to raise \$1500 to have \$500 left over after paying for food, which can be achieved if $\frac{\$1500}{\$5 \text{ per person}} = \boxed{300}$ people attend his party.

34. How many integers from 1 to 100 (inclusive) leave a remainder of 2 or 4 when divided by 5?

Solution: When divided by 5, a number can only have a remainder of 0, 1, 2, 3, or 4. These remainders follow a pattern: 1, 2, 3, 4, 0, 1, 2, 3, 4, 0, etc. For every five consecutive numbers, two of them will have remainders of 2 or 4. There are 100 numbers from 1 to 100, so we can split this up into 20 sets of five consecutive numbers each. In each set, there are two numbers that have remainders of 2 or 4, so there are $20 \cdot 2 = \boxed{40}$ total numbers with this property.

35. If one Jenny equals two Kristinas, three Kristinas equal six Lisas, and ten Lisas equal fifteen Margoes, how many Jennies equal 180 Margoes?

Solution: 1J = 2K. 3K = 6L, so diving both sides by 3, we get K=2L. 10L = 15M, so dividing both sides by 5, we get 2L=3M. So 1J = 2K = 2(2L) = 2(3M) = 6M. So 180M = 30(6M) = 30J, so $\boxed{\text{30 Jennies}}$ equal 180 Margoes.

36. The area of a square is equal to its perimeter. What is its side length?

Solution: Let s be the side length of the square. The area of the square is s^2. The perimeter of the square is $4s$. So we have $s^2 = 4s$. Dividing both sides by s, we get $s = \boxed{4}$.

37. What is last digit of 2^{16}?

Solution 1: By brute force, $2^{16} = 65536$, so the last digit is $\boxed{6}$.

Solution 2: Write out the powers of two starting with 2^1. Notice that the last digits form the sequence $2, 4, 8, 6, 2, 4, 8, 6, \ldots$ and that every fourth power of two has 6 as its last digit. Therefore, $\boxed{6}$ is the last digit of 2^{16}.

38. Ian is filling in a sequence with a specific pattern: 1, 2, 3, 6, 11, 20, 37, 68, a, b. Compute $a - b$.

Solution: Each number in the sequence (after the first three) is the sum of the three previous numbers. So $b = 68 + 37 + a = 105 + a$. Thus, $a - b = a - (105 + a) = \boxed{-105}$.

39. Jordan is solving a calculus problem that requires him to give a single integer answer. Let p be the probability he gets the problem right and q be the probability he gets the problem wrong. Assuming that there is no partial credit, what is $p + q$?

Solution: If Jordan does not get the problem right, he gets it wrong, and vice versa. The total probability is 1 and the probability that Jordan gets the problem right is p, so the probability that he gets the problem wrong is $1 - p$, which equals q. Thus, $p + q = p + (1 - p) = \boxed{1}$.

40. Jordan buys 5 Häagen Dazs ice creams and 3 steaks for \$9. Bryan buys 13 Häagen Dazs ice creams and 15 steaks for \$27. If Fahmid buys one Häagen Dazs ice cream and one steak, how much does he pay?

Solution: Let h be the cost of ice cream and s be the cost of steak (both in dollars). Then the first statement translates to $5h + 3s = 9$ and the second statement translates to $13h + 15s = 27$. Add up the equations to get $18h + 18s = 36$ and divide both sides by 18 to get the cost of one Haagen Dazs and one steak, which is $\boxed{\$2}$.

41. Kevin's book has 707 pages. How many sixes will he come across in the page numbers?

Solution: In the ones digit, a 6 will appear once every 10 pages. So in the first 700 pages, there will be 70 6's in the ones column. There is one more for page 706, so in total there are 71 6's in the ones column. In the tens column, there will be 10 6's every 100 pages (pages 60 through 69, for example). So in the first 700 pages, there will be 70 6's in the tens column. Pages 701-707 all have 0's in the tens column, so there will be 70 6's in the tens column. In the hundreds column, there will be 6's from pages 600-699 only. Thus, there are 100 6's in the hundreds column. Adding these together, we see that there are $71 + 70 + 100 = \boxed{241}$ total 6's in the page numbers.

42. If I have twenty-five meters of wire and want to use it to make a rectangle with the biggest possible area, what is the area of that rectangle in square meters?

Solution: With a given perimeter, the rectangle with the largest area is a square. There are 25 meters of wire to make four sides, so each side has a length of $\frac{25}{4}$. Thus, the area of the square is $\left(\frac{25}{4}\right)^2 = \boxed{\frac{625}{16}}$.

43. Find $\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + \cdots + \frac{1}{99 \cdot 100}$.

Solution: $\frac{1}{n(n+1)} = \frac{(n+1)-n}{n(n+1)} = \frac{n+1}{n(n+1)} - \frac{n}{n(n+1)} = \frac{1}{n} - \frac{1}{n+1}$. Using this, we can rewrite the expression as $\left(\frac{1}{1} - \frac{1}{2}\right) + \left(\frac{1}{2} - \frac{1}{3}\right) + \left(\frac{1}{3} - \frac{1}{4}\right) + \cdots + \left(\frac{1}{99} - \frac{1}{100}\right)$. Canceling out the middle terms, we are left with $\frac{1}{1} - \frac{1}{100} = \frac{100}{100} - \frac{1}{100} = \frac{100-1}{100} = \boxed{\frac{99}{100}}$.

44. Kevin has two red balls, three blue balls, and one white ball. All balls of the same color are indistinguishable. How many ways are there of arranging the six balls in a line?

Solution: There are $6! = 720$ possible ways to arrange 6 balls in a line. However, since some of the balls used are the same, some of these arrangements will be identical. There are 2 red balls used, so the total number of possibilities must be divided by $2! = 2$ to account for repeats. Similarly, there are 3 blue balls used, so the total number of possibilities must be divided by $3! = 6$ to account for repeats. Therefore, there are $\frac{720}{2 \cdot 6} = \boxed{60}$ possible arrangements.

45. Evaluate $(-0)+(-1)-(-2)+(-3)-(-4)+(-5)-(-6)+(-7)-(-8)+(-9)-(-10)+...+(-99)-(-100)$.

Solution: Group the numbers into 50 pairs: $(-1+2)+(-3+4)+\cdots+(-99+100)$. Each pair simplifies to 1, so the total sum is $50 \times 1 = \boxed{50}$.

46. Find the area of the triangle bounded by the x-axis, the y-axis, and the line $y = -7x + 14$.

Solution: The line $y = -7x+14$ intersects the axes at the points $(2, 0)$ and $(0, 14)$. The region formed is a right triangle with a height of 14 and a base of 2, so the area is $\frac{1}{2} \cdot (14) \cdot (2) = \boxed{14}$.

47. A man is born in 1900 and dies in 2000. He is not 100 years old when he dies. How old is he?

Solution: In the man already had his birthday in 2000, he would be 100 years old. If he did not already have his birthday in 2000, he would be one year less, making him $\boxed{99}$ years old.

48. Yi is trying to guess the locker combination to a lock. The lock code is a 3 digit number, with the digits from 0 to 9. He knows that the first digit is not a 9 and the second digit is not a 9. He also knows that the product of the digits is 0. What is the maximum possible sum of the digits of the locker code? (The first digit can be zero)

Solution: Since the product is 0, at least one of the digits is 0; since we are trying to maximize the sum, there will only be one 0, and the rest of the digits will be as large as possible. If the 0 is the first digit, then the second digit can be at most 8, and the last digit can be at most 9, with a sum of 17. A similar situation occurs if the 0 is the second digit. If the 0 is the last digit, then both the first and second digits can be at most 8, with a sum of 16. Therefore, the maximum possible sum is $\boxed{17}$.

49. Bob is driving down a straight road in his car. He drives for two hours at 45 miles per hour and then for three more hours at 40 miles per hour. How far, in miles, does Bob travel in total?

Solution: distance = rate $\cdot$ time. In the first part of his trip, Bob travels $2 \cdot 45 = 90$ miles. In the second part of his trip, Bob travels $3 \cdot 40 = 120$ miles. Thus, Bob travels $90 + 120 = \boxed{210 \text{ miles}}$ in total.

50. If I reverse the digits in a two-digit number, the number increases by 18. What is the greatest possible sum of the new number and the old number?

Solution: Let the two-digit number be $\underline{A}\,\underline{B}$, where A is the tens digit and B is the ones digit. Then the two-digit number is equal to $10A + B$. Reversing the digits of the number gives $\underline{B}\,\underline{A}$, where B is the tens digit and A is the ones digit. This number is equal to $10B + A$. We know that $\underline{A}\,\underline{B} + 18 = \underline{B}\,\underline{A}$, so $(10A + B) + 18 = (10B + A)$. Subtracting A and B from both sides, we get $9A + 18 = 9B$. Dividing both sides by 9, we get $A + 2 = B$. A and B are digits, which means that they can be at most 9. Therefore, the largest digits satisfying $A+2 = B$ are $A = 7$ and $B = 9$. Then the old number is 79 and the new number is 97, so the sum is $79 + 97 = \boxed{176}$.