

# Bergen County Academies Math Competition - 5th Grade

## General Rules

- Calculators are not allowed.
- This is an individual test, so you may not communicate with anyone else taking it.
- Once time begins, we will not answer any questions about the problems.
- You will have 90 minutes to solve 50 problems. Once time is called, you must put down your pen or pencil and stop working.
- Scores will be posted on the website within a couple of days. Your score will appear next to your identification number.

## Specifics

- You may use space on your test paper and additional scrap paper to do work. Your answers must be written on the answer sheet. We will not look at answers written on your test paper.
- Each problem has only one answer. If you put more than one answer for a problem, you will be marked wrong. When changing an answer, be sure to erase or cross out completely.
- Write legibly. If the graders cannot read your answer, it will be marked incorrect.
- Fractions should be written in lowest terms. For example, if the answer is  $\frac{1}{2}$ , then  $\frac{2}{4}$  will not be accepted although the two fractions are numerically equal.
- All other answers should be written in simplest form.
- If a unit is indicated in the problem, the answer must be given in that unit. For instance, if the problem asks for the answer in hours, you cannot give your answer in minutes. Furthermore, you don't need to write the unit, as the graders will assume your answer is in the units asked for in the problem.
- There is no penalty for guessing.
- Ties will be broken based on the number of correct responses to the last ten questions. If a tie remains, then the correct responses to the last five questions will break the tie.
- We will announce how much time is remaining often during the test.

1. Find  $2 + 5 \times 6$ .
2. Compute  $\frac{666666}{333333}$ .
3. What is the average (arithmetic mean) of .4, .04, .004, and .0004?
4. Chef J reaches into a chocolate storage bin, which houses gratuitous amounts of two types of chocolate: dark chocolate and white chocolate. If he blindly grabs pieces of chocolate from the storage bin, how many must he grab to ensure he has at least three pieces of the same type of chocolate?
5. How many different 4 digit numbers can be formed by rearranging the digits of 2011? (Numbers starting with 0, e.g., 0121 and 0211, do not count).
6. Harry Potter is mixing potions in Potions class. He is supposed to add  $2\frac{1}{2}$  cups of newt scales,  $\frac{1}{8}$  cup of unicorn blood,  $\frac{11}{4}$  cups of treacle tart, and  $\frac{2}{3}$  cups of phoenix tears. Assuming he does this correctly, what will be the final volume of the potion?
7. Compute  $11^4$ .
8. What is the remainder when 12345678 is divided by 3?
9. Evaluate  $(-10) \times (-9) \times \dots \times (9) \times 10$ .
10. Michael flips heads with a fair coin 50 times in a row. What is the probability that after the next flip, the coin will again turn up heads?
11. It takes 3 rabbits 3 minutes to eat 3 carrots. How many minutes does it take for 6 rabbits to eat 6 carrots?
12. A palindrome is a number that reads the same back and forth. For example, 12321 is a palindrome. Find the number of 6-digit palindromes (palindromes cannot start with the digit 0.)
13. One parasprite spawns a new parasprite every 20 minutes. If we start out with one parasprite, how many parasprites will there be after two hours?
14. Dumbledore is eating jelly beans. In a bag of jelly beans, there are 4 blue jelly beans, 9 red jelly beans, and 7 black jelly beans. If Dumbledore randomly picks three jelly beans and eats each one right after taking it out of the bag, then what is the probability that none of the three jelly beans he chose are blue?
15. Which of the following numbers is the largest?  $2.2, 2\frac{1}{3}, \frac{12}{5}, \frac{1}{2.2}$ ?
16. If there are 17 Sickles in a Galleon, and 29 Knuts in a Sickle, how many Knuts are there in 2 Galleons and 2 Sickles?
17. A cylinder has surface area  $10\pi$  and height 4. What is its volume?
18. Find the number halfway between  $\frac{1}{3}$  and the number halfway between  $\frac{1}{2}$  and  $\frac{1}{3}$ .
19. Compute  $2 + 4 + 6 + 8 + 10 + 12 + 14 + 16 + 18 + 20 - 1 - 3 - 5 - 7 - 9 - 11 - 13 - 15 - 17 - 19$ .

20. Find the area of a triangle with side lengths 6, 8, and 10.
21. If  $8^{3x+4} = 16^{5x}$ , find  $x$ .
22. A hemisphere of radius 5 is glued to the top of a cylinder with radius 5 and height 10. Find the surface area of the resulting solid.
23. A palindrome is a number that reads the same back and forth. Find the number of 8-digit palindromes whose digits sum to 15.
24. Suppose  $a, b, c, d$  are distinct digits such that the three-digit numbers  $7\bar{a}9$  and  $2\bar{b}4$  sum to the four-digit number  $\overline{cd}13$ . Find all possible values of  $a + b + c + d$ .
25. Alex the Zookeeper is giving a polar bear a haircut. If the bear's surface area is 180 square feet, and Alex finishes the haircut in 2 hours, then what is Alex's average rate of haircutting in square inches per minute?
26. In Mr. Kim's Pizzeria, I can choose up to 3 of 12 possible toppings. How many different topping combinations can I order on my pizza?
27. Find the third smallest integer with exactly 3 divisors.
28. In an excellent school somewhere far away, there are 200 students. Given that 73 students take Pig Latin, 98 students take Klingon, and 18 students take both, how many students take neither Pig Latin nor Klingon?
29. Michael is trying to guess JP's favorite number. Michael is told that it is a positive integer less than 100, that two more than the value is a multiple of 6, that the sum of its digits is a multiple of 7, and that it is a multiple of 8. Find JP's favorite number.
30. Let  $P(x) = x^2 - 8x + 19$ . Find  $P(P(P(P(P(P(P(P(P(3))))))))))$ .
31. Find the last digit of  $9^{8^{7^{6^{5^{4^{3^{2^1}}}}}}}$ .
32. Find all real  $x$  such that  $x^3 + x^2 + x + 1 = 0$ .
33. What day of the week will October 17, 2012 be?
34. What is the smallest positive integer  $n$  greater than 1 such that  $n^2$  is a cube and  $n^3$  is a square?
35. A snail is at the bottom of a long tube that measures 20 meters. On a given day, the snail will travel 3 meters up the tube, and during the night it will sink 1 meter down the tube. On what day will the snail finally reach the top of the long tube?
36. Find the 200th term in the following sequence: 1, 2, 2, 3, 3, 3, 4, 4, ...
37. Let  $\Gamma$  be a circle centered at  $O$ . Given that  $AB$  is a chord of length 12 in the circle, and the distance from  $O$  to  $AB$  is 3, compute the area of  $\Gamma$ .
38. Evaluate  $2 - 1 + \frac{1}{2} - \frac{1}{4} + \frac{1}{8} - \frac{1}{16} + \dots$ .
39. Find the number of diagonals in a regular dodecagon.
40. Find  $1.9\overline{8}$  (that is, 1.9888... with repeating 8's) in simplest fractional form.

41. Find the smallest positive integer that cannot be represented as the sum of three not necessarily distinct perfect squares.
42. A regular fair six-sided die is rolled twice. What is the probability that the first number rolled divides the second number rolled?
43. In  $\triangle ABC$ ,  $AB = 4$ ,  $BC = 5$ ,  $CA = 6$ , and the bisector of angle  $A$  intersects  $BC$  at  $D$ . Find the length of  $BD$ .
44. Find the real values of  $x$  that satisfy the equation  $x^4 + 4x^3 + 6x^2 + 4x - 15 = 0$ .
45. A rectangle is inscribed in a circle. If the length and width of the rectangle are 24 and 10, respectively, what is the area of the circle?
46. Steven randomly draws two cards from a fair deck of 52 cards. If he picks a 6 and a 7, what is the probability that after he picks up another card, the sum of the values of his cards does not exceed 21? (Aces count as 1, and all face cards count as 10).
47. There are 5 distinct balls. If you pick two balls at random and Bob picks three balls at random, what is the probability that one of your balls will be among one of Bob's balls?
48. Applejack, Twilight Sparkle, and Rainbow Dash are picking apples at Sweet Apple Acres. If it takes Applejack and Twilight Sparkle 3 days to pick all the apples, Applejack and Rainbow Dash 4 days to pick all the apples, and Twilight Sparkle and Rainbow Dash 6 days to pick all the apples, how long would it take all three of them together to pick apples?
49. How many integers  $n$  between 1 and 2011, inclusive, have the property that  $n^2 + 2n + 3$  is divisible by 3?
50. Let  $A_1A_2A_3 \dots A_{20000}$  be a regular polygon with 20,000 sides. If  $A_1A_{10001} = 20$ , find the integer closest to the area of  $A_1A_2A_3 \dots A_{20000}$ .