# Bergen County Academies Math Competition - 8th Grade

## General Rules

- Calculators are not allowed.
- This is an individual test, so you may not communicate with anyone else taking it.
- Once time begins, we will not answer any questions about the problems.
- You will have 90 minutes to solve 50 problems. Once time is called, you must put down your pen or pencil and stop working.
- Scores will be posted on the website within a couple of days. Your score will appear next to your identification number.

## Specifics

- You may use space on your test paper and additional scrap paper to do work. Your answers must be written on the answer sheet. We will not look at answers written on your test paper.
- Each problem has only one answer. If you put more than one answer for a problem, you will be marked wrong. When changing an answer, be sure to erase or cross out completely.
- Write legibly. If the graders cannot read your answer, it will be marked incorrect.
- Fractions should be written in lowest terms. For example, if the answer is $\frac{1}{2}$, then $\frac{2}{4}$ will not be accepted although the two fractions are numerically equal.
- All other answers should be written in simplest form.
- If a unit is indicated in the problem, the answer must be given in that unit. For instance, if the problem asks for the answer in hours, you cannot give your answer in minutes. Furthermore, you don't need to write the unit, as the graders will assume your answer is in the units asked for in the problem.
- There is no penalty for guessing.
- Ties will be broken based on the number of correct responses to the last ten questions. If a tie remains, then the correct responses to the last five questions will break the tie.
- We will announce how much time is remaining often during the test.

1. Compute $\frac{666666}{333333}$.

2. What is the average (arithmetic mean) of .4, .04, .004, and .0004?

3. Jason has \$93 but he wants to have prime number of dollars. What is the least amount of money he needs to spend in order to have a prime number of dollars?

4. Find the median of the following set of numbers: 1, 6, 2, 3, 2, 9, 8, 5.

5. Compute $(-3)^{-3} - 3$.

6. Two variables are called *inversely proportional* if their product is constant. If $x^2$ and $y$ are inversely proportional, and $y = 4$ when $x = 6$, find $y$ when $x = 4$.

7. Which of the following numbers is the largest? $2.2, 2\frac{1}{3}, \frac{12}{5}, \frac{1}{2.2}$?

8. Find the number halfway between $\frac{1}{3}$ and the number halfway between $\frac{1}{2}$ and $\frac{1}{3}$.

9. Let $f(x) = x + 1$ and $g(x) = 2x$. Find $f(g(f(g(f(g(1))))))$.

10. Find the area of a triangle with side lengths 6, 8, and 10.

11. In how many ways can 4 people arrange themselves in a line?

12. Groovy-Band sold $\frac{11}{12}$ of the available tickets for their latest show, and total ticket sales were \$93,500. What would have been the total ticket sales, in dollars, if Groovy-Band had sold all of the available tickets?

13. If there are 17 Sickles in a Galleon, and 29 Knuts in a Sickle, how many Knuts are there in 2 Galleons and 2 Sickles?

14. One parasprite spawns a new parasprite every 20 minutes. If we start out with one parasprite, how many parasprites will there be after two hours?

15. If $\frac{49^{27x}}{7^{9x}} = 49$, find $x$.

16. A hemisphere of radius 5 is glued to the top of a cylinder with radius 5 and height 10. Find the surface area of the resulting solid.

17. If you roll three dice, what is the probability that the product of the numbers on the dice is odd?

18. When Dudley Dursley turned 9, his parents gave him $n$ presents, where $n$ has 9 different positive divisors. What is the smallest possible value of $n$?

19. Mike Sun is tethered by a rope of length 4 meters to the corner of a building. This corner is a right angle, and the sides of the building extend 30 meters in each direction. Find the area which Mike can roam.

20. Find all integers $n$ such that $\frac{n(2+2^n)}{n+1} = 2n$.

21. How many arrangements are there of the letters in the word "ANAGRAM"?

22. Find the 200th term in the following sequence: $1, 2, 2, 3, 3, 3, 4, 4, \ldots$

23. If $x^3 - 3x^2 - 4x = 30$ and $x^3 - 6x^2 + 4x = -5$, find $x$.

24. Find $1.9\overline{8}$ (that is, 1.9888... with repeating 8's) in simplest fractional form.

25. What day of the week will October 17, 2012 be?

26. Find the last digit of $9^{8^{7^{6^{5^{4^{3^{2^{1}}}}}}}}$.

27. A regular fair six-sided die is rolled twice. What is the probability that the first number rolled divides the second number rolled?

28. James has 200 feet of fence to build a rectangular enclosure around a house which has one side on the river. This means that the house needs only to be covered on three of its sides. What is the largest possible area that the fence can enclose?

29. Find the number of zeroes at the end of $((3!)!)!$, where $n! = n \times (n-1) \times \cdots \times 2 \times 1$.

30. Find the quadratic equation whose coefficient of $x^2$ is 1, and whose roots are the squares of the roots of $x^2 + 4x - 2$.

31. Let $\triangle ABC$ be an equilateral triangle of side length 4, and let $A_1, B_1, C_1$ be the midpoints of segments $BC$, $CA$, and $AB$, respectively. Let $A_2, B_2, C_2$ be the midpoints of segments $B_1C_1$, $C_1A_1$, and $A_1B_1$, respectively. Find the ratio of the area of $\triangle A_2B_2C_2$ to the area of $\triangle ABC$.

32. Compute $1^2 - 2^2 + 3^2 - 4^2 + \ldots - 50^2 + 51^2$.

33. Find all integers $x$ such that $x^2 + 2x - 8$ is a prime number.

34. In $\triangle ABC$, $AB = 4$, $BC = 5$, $CA = 6$, and the bisector of angle $A$ intersects $BC$ at $D$. Find the length of $BD$.

35. If $x - \frac{1}{x} = 3$, find $x^2 + \frac{1}{x^2}$.

36. Let $r, s, t$ be the roots of the cubic $x^3 - 6x^2 + 5x + 1$. Find $(2-r)(2-s)(2-t)$.

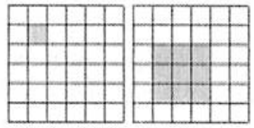
37. Steven randomly draws two cards from a fair deck of 52 cards. If he picks a 6 and a 7, what is the probability that after he picks up another card, the sum of the values of his cards does not exceed 21? (Aces count as 1, and all face cards count as 10).

38. Two real numbers between 0 and 1 are chosen randomly. Find the probability that their sum is less than $\frac{1}{2}$.

39. There are 5 distinct balls. If you pick two balls at random and Bob picks three balls at random, what is the probability that one of your balls will be among one of Bob's balls?

40. How many integers $n$ between 1 and 2011, inclusive, have the property that $n^2 + 2n + 3$ is divisible by 3?

41. Find the number of squares that can be formed by the $1 \times 1$ squares on a $6 \times 6$ chessboard. (The diagram shows examples of some squares that can be formed.)



42. Find $11_2 + 11_3 + 11_4 + \cdots + 11_{100}$. ($11_b$ is taken to mean 11 in base $b$.)

43. Let $A_1A_2A_3 \ldots A_{20000}$ be a regular polygon with 20,000 sides. If $A_1A_{10001} = 20$, find the integer closest to the area of $A_1A_2A_3 \ldots A_{20000}$.

44. Applejack, Twilight Sparkle, and Rainbow Dash are picking apples at Sweet Apple Acres. If it takes Applejack and Twilight Sparkle 3 days to pick all the apples, Applejack and Rainbow Dash 4 days to pick all the apples, and Twilight Sparkle and Rainbow Dash 6 days to pick all teh apples, how long would it take all three of them together to pick apples?

45. Find the largest integer $a$ such that $a^2$ can be written as the sum of a perfect square and a prime number less than 100.

46. Find the volume of the region defined by $3x + 4y + 5z \leq 60$ and $x, y, z \geq 0$.

47. If $x - \frac{1}{x} = 3$, find $x^2 + \frac{1}{x^2}$.

48. Find the sum of the digits of $(10^{100} - 1) - \frac{10^{100} - 1}{10^{20} - 1}$.

49. Given that $929 = 23^2 + 20^2$, and that $2 \cdot 929$ can be expressed in the form $c^2 + d^2$ for some positive integers $c$ and $d$, find $c + d$.

50. Given that $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \cdots = \frac{\pi^2}{6}$, find $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \cdots$.