1 Grade 5 Solutions

1. $2 + 5 \cdot 6 = 2 + 30 = \boxed{32}$.

2. The answer is $\boxed{2}$.

3. $\dfrac{0.4 + 0.04 + 0.004 + 0.0004}{4} = \dfrac{0.4444}{4} = \boxed{0.1111}$.

4. In the worst case scenario, Chef J grabs 2 chips and 2 truffles. However, whichever chocolate he picks next, there are going to be at least 3 pieces of the same type of chocolate. Therefore, the answer is $\boxed{5}$.

5. There are $\boxed{9}$ such numbers. We can find them simply by listing them all out: 2011, 2101, 2110, 1201, 1210, 1120, 1102, 1012, and 1021.

6. The final volume of the potion is the sum of the volumes of the ingredients, so we wish to find $2\frac{1}{2} + \frac{1}{8} + \frac{11}{4} + \frac{2}{3}$. To find this sum, we first write $2\frac{1}{2}$ as a mixed fraction: $\frac{5}{2}$. We then put it all over a common denominator, as follows: $\frac{5}{2} + \frac{1}{8} + \frac{11}{4} + \frac{2}{3} = \frac{60}{24} + \frac{3}{24} + \frac{66}{24} + \frac{16}{24} = \boxed{\frac{145}{24}}$.

7. $11^4 = 11^2 \cdot 11 \cdot 11 = 121 \cdot 11 \cdot 11 = 1331 \cdot 11 = \boxed{14641}$.

8. We apply the divisibility rule for 3. The sum of the digits of 12345678, which is 36, is divisible by 3, so 12345678 is itself divisible by 3. Thus, it leaves a remainder of $\boxed{0}$ when divided by 3.

9. Since 0 appears in this product, the product of these numbers will be $\boxed{0}$.

10. Since the coin is fair, the answer is $\boxed{\frac{1}{2}}$. (The fact that he flipped 50 heads in a row before is irrelevant.)

11. Split the 6 rabbits into two groups of three rabbits. In three minutes, each of the two groups of rabbits willl have eaten 3 carrots, so after $\boxed{3 \text{ minutes}}$, they will have eaten 6 carrots in total.

12. Let the palindrome be $abccba$. There are 9 choices for a (1-9), and 10 choices each for b and c (0-9). This gives a total of $9 \cdot 10 \cdot 10 = \boxed{900}$ 6 -digit palindromes.

13. After every 20 minutes, the number of parasprites doubles: each of the original parasprites remains, and each of them spawns a new parasprite. Since six sets of 20 minutes pass in two hours, there will be a total of $2^6 = \boxed{64}$ parasprites after 2 hours.

14. There are 20 jelly beans in total, with 4 of them blue. The probability of the first not being blue is $\frac{16}{20}$, the probability of the second not being blue is $\frac{15}{19}$, and the probability of the third not being blue is $\frac{14}{18}$. Thus the probability of getting three non-blue jelly beans is $\frac{16}{20} \cdot \frac{15}{19} \cdot \frac{14}{18} = \boxed{\frac{28}{57}}$.

15. $\frac{12}{5}$ is equal to 2.4. $2\frac{1}{3}$ is equal to 2.33...., which is clearly less than 2.4. Also, $\frac{1}{2.2} = 1 < 2.4$, and clearly $2.2 < 2.4$. Thus, $\boxed{\frac{12}{5}}$ is the largest of these numbers.

16. Since a Galleon is worth 17 Sickles, 2 Galleons are worth 34 Sickles. Thus, 2 Galleons and 2 Sickles are worth 36 Sickles. Since there are 29 Knuts in a Sickle, the answer is $36 \cdot 29 = \boxed{1044}$ Knuts.

17. The surface area of a cylinder is $2\pi rh + 2\pi r^2$. When $h = 4$, we can solve to find that $r = 1$. Thus the volume is $\pi r^2 h = \boxed{4\pi}$.

18. The number halfway between $\frac{1}{2}$ and $\frac{1}{3}$ is $\frac{\frac{1}{2}+\frac{1}{3}}{2} = \frac{\frac{3}{6}+\frac{2}{6}}{2} = \frac{5}{12}$. The number halfway between $\frac{1}{3}$ and $\frac{5}{12}$ is $\frac{\frac{1}{3}+\frac{5}{12}}{2} = \frac{\frac{4}{12}+\frac{5}{12}}{2} = \frac{9}{24} = \boxed{\frac{3}{8}}$.

19. Note that we can rearrange the expression to $(2-1)+(4-3)+(6-5)+\cdots+(18-17)+(20-19) = \underbrace{1+1+\cdots+1}_{10} = \boxed{10}$.

20. A triangle with side lengths of 6, 8, and 10 is a right triangle with legs of lengths 6 and 8 and a hypotenuse of length 10 because the triangle satisfies the Pythagorean Theorem ($6^2 + 8^2 = 10^2$.) The area is therefore half the product of the legs: $\frac{1}{2} \times 6 \times 8 = \boxed{24}$.

21. Rewrite this with each of the bases as 2. $8^{3x+4} = 2^{3(3x+4)} = 2^{9x+12}$ and $16^{5x} = 2^{4(5x)} = 2^{20x}$. Equating exponents, we have $20x = 9x + 12 \implies \boxed{x = \frac{12}{11}}$.

22. The resulting figure has one circular face, of surface area πr^2, one lateral surface of a cylinder, with surface area $2\pi rh$, and one-half of a sphere, which has surface area $\frac{1}{2}(4\pi r^2) = 2\pi r^2$. Thus, the total surface area is $3\pi r^2 + 2\pi rh = 3\pi(5^2) + 2\pi(5)(10) = \boxed{175\pi}$.

23. There are no such palindromes. Suppose, for the sake of contradiction there exists a palindrome with 8 digits and whose digits sum to 15 be *abcddcba*. Then we have that $2(a+b+c+d) = 15$, which is impossible since 15 is even, so our answer is $\boxed{0}$.

24. We have $7a9 + 2b4 = cd13$. Since there is a carry in the ones digit, we must have either $a+b+1 = 1$ or $a+b+1 = 11$. In the former case, $7a9+2b4 < 1000$, which is impossible since $cd13$ is a four-digit number. Thus, $a+b = 11$, whence c and d must be 1 and 0, respectivelly, so our answer is $10+1+0 = \boxed{11}$.

25. Cutting 180 square feet in 2 hours is equivalent to cutting 90 square feet in 1 hour. Convert this to inches per second by realizing that there are $12 \cdot 12 = 144$ square inches in a square foot and 60 minutes in an hour to get a total of $\frac{144 \cdot 90}{60} = \boxed{216}$ square inches per second.

26. Since the person can choose *up* to 3 toppings out of 12, the different combinations of toppings include a pizza with 3 toppings, 2 toppings, 1 topping, and 0 toppings . The total possible number of pizzas with 3 toppings is $\binom{12}{3} = 220$; the total number of pizzas with 2 toppings is $\binom{12}{2} = 66$; the total number of pizzas with 1 topping is $\binom{12}{1} = 12$; and the number of

pizzas with no toppings is 1. Thus, our answer is $220 + 66 + 12 + 1 = \boxed{299}$. (*Note:* Because of a slight ambiguity with the phrase "combination of toppings," which may seem to imply that at least one topping must be used, we accept an answer of 298 as well.)

27. A number has exactly 3 divisors if and only if it is the square of a prime number. The third smallest number of this form is the square of the third smallest prime, 5, so the answer is $5^2 = \boxed{25}$.

28. If 73 people take Pig Latin, 98 take Klingon, and 18 take both, then 80 take just Klingon, and 55 take just Pig Latin, so a total of $80 + 55 + 18 = 153$ people take at least one language. Thus, if there are 200 students in all, $200 - 153 = \boxed{47}$ take no language.

29. We know that JP's favorite number is a two digit number since the number is less than 100. The sum of the digits is a multiple of 7, so the sum of the digits can either be 7 or 14. The numbers that satisfy these conditions are $59, 68, 77, 86, 95, 16, 25, 34, 43, 52, 61$, and 70. Since the number is also a multiple of 8, and the only one of these numbers satisfying this is 16, our answer must be $\boxed{16}$.

30. Let $P^k(n)$ denote P iterated on n k times. We wish to find $P^9(3)$. Since $P(3) = 4$ and $P(4) = 3$, we have $P^2(3) = 3$. Therefore, $P^9(3) = P^7(3) = P^5(3) = P^3(3) = P^1(3) = \boxed{4}$.

31. The number 9^n ends with 1 if n is even, and ends with 9 if n is odd. Since $8^{7^{6^{5^{4^{3^{2^{1}}}}}}}$ is even, the last digit of $9^{8^{7^{6^{5^{4^{3^{2^{1}}}}}}}}$ is $\boxed{1}$.

32. By inspection, $x = -1$ satisfies the equation. Factoring, we find that $x^3 + x^2 + x + 1 = (x + 1)(x^2 + 1)$. $x^2 + 1$ has no real roots, so the only real x that satisfies the equation is $x = \boxed{-1}$.

33. First of all, we must consider that 2012 is a leap year so we must consider the extra day (hence 2012 will have 366 days instead of 365). We must also consider that October 16, 2011, the day of the contest, is a Sunday. One year from now, which is 366 days later, October 16, 2012 will be in 52 weeks and 2 days, so October 16 will be a Tuesday. Therefore, October 17 will be a $\boxed{\text{Wednesday}}$.

34. If n^2 is a cube, n must also be a cube; if n^3 is a square, then n must also be a square. Thus, n must be both a square and a cube, so it must be a sixth power. The smallest sixth power larger than 1 is $2^6 = \boxed{64}$.

35. After a cycle of one day and one night, the snail has a net movement of 2 meters up the tube. After the ninth day, the snail is $8 \cdot 2 + 3 = 19$ meters from the bottom of the tube. During the ninth night, the snail sinks 1 meter down, and reaches the top of the 20 meter tube on the tenth day. Therefore, the answer is $\boxed{10}$.

36. It is easy to see that the number n appears between positions $(1 + 2 + \cdots + (n - 1)) + 1$ and $(1 + 2 + \cdots + n)$ inclusive, that is, $\frac{n(n-1)}{2} + 1$ and $\frac{n(n+1)}{2}$ inclusive. Since 200 lies between $190 = \frac{20 \cdot 19}{2} + 1$ and $210 = \frac{20 \cdot 21}{2}$ inclusive, our answer is $\boxed{20}$.

37. After drawing the figure, we find that half the chord (which has a length of 6), the radius of the circle, and the line from O to AB form a right triangle with legs 3 and 6. Applying

the Pythagorean Theorem, we find that the hypotenuse is $3\sqrt{5}$, which is also the radius. The area is therefore $\pi \cdot \text{radius}^2 = \pi\left(3\sqrt{5}\right)^2 = \boxed{45\pi}$.

38. Grouping the terms together, we have $(2-1)+(\frac{1}{2}-\frac{1}{4})+(\frac{1}{8}-\frac{1}{16})+\cdots = 1+\frac{1}{4}+\frac{1}{16}+\cdots$. Let $S = 1+\frac{1}{4}+\frac{1}{16}+\cdots$. Multiply both sides by 4 to obtain $4S = 4+1+\frac{1}{4}+\frac{1}{16}+\cdots$. Substituting this in, we get $4S = 4+S$, so $S = \boxed{\frac{4}{3}}$.

39. In a given convex polygon with n sides, there are $\frac{n(n-3)}{2}$ diagonals.To see this, notice that we can count the number of diagonals by finding how many pair of vertexes that are not on the same side that the polygon has. There are n choices for the first point. For the second point, we cannot use the same point and we cannot use the points immediately adjacent to the first point, as explained above. Thus there are $n-3$ choices for the second point. We count each diagonal twice through this method (the first and second points can be switched around) so we need to divide this number by 2 to figure out the number of diagonals. Hence, in a dodecagon, which is a polygon with 12 sides, there are $\frac{12\cdot 9}{2} = \boxed{54}$ diagonals.

40. $1.9\overline{8} = \frac{19+0.\overline{8}}{10} = \frac{19+\frac{8}{9}}{10} = \frac{171+8}{90} = \boxed{\frac{179}{90}}$.

41. Trial and error gives that $\boxed{7}$ is the smallest positive integer that cannot be represented as the sum of three not necessarily distinct perfect squares.

42. If the first roll is a 1, we have six choices for the second roll. If it is a 2, that gives us three choices for the second roll (2, 4, 6). If it is a 3, then there are only two choices (3, 6), and for 4, 5, and 6, there is only one choice (itself). That gives us a probability of $\frac{6+3+2+1+1+1}{36} = \boxed{\frac{7}{18}}$.

43. Let $x = BD$ and $y = DC$. Clearly, $x+y = 5$. Also, by the angle bisector theorem, $\frac{x}{y} = \frac{4}{6} = \frac{2}{3}$, so $3x = 2y = 2(5-x)$, so $5x = 10$, so $x = BD = \boxed{2}$.

44. Note that we can rewrite $x^4+4x^3+6x^2+4x-15 = 0$ as $x^4+4x^3+6x^2+4x+1 = 16$ by adding 16 to both sides. Note that the left-hand side can be factored into $(x+1)^4 = 16$. We take the fourth root of both sides, making sure to consider only real values, so $x+1$ must be either 2 or -2. Therefore the only real values for x satisfying the equation are $\boxed{1}$ and $\boxed{-3}$.

45. Using the Pythagorean Theorem on the diagonal of the rectangle gives us that it has length $\sqrt{10^2+24^2} = 26$. Since the diagonal is the hypotenuse of a right triangle inscribed in a circle, it is also a diameter. Thus the radius is 13 and its area is $13^2\pi = \boxed{169\pi}$.

46. There are 50 cards from which Steven can draw. The cards he can draw to keep from going above 21 are each of the four aces, twos, threes, fours, and fives, the three remaining sixes, the three remaining sevens, and the four eights. There are 30 such cards in total, so the probability is $\frac{30}{50} = \boxed{\frac{3}{5}}$.

47. Consider the 3 balls Bob picks. The only way one of your balls will not be among Bob's balls is if you pick the two he did not pick. This only happens with probability $\frac{1}{\binom{5}{2}} = \frac{1}{10}$, so the probability one of your balls will be among Bob's chosen ones is $1 - \frac{1}{10} = \boxed{\frac{9}{10}}$.

48. Let a, t, r be the percentage of the apples that Applejack, Twilight Sparkle, and Rainbow Dash can pick in one day, respectively. We are given that $3(a+t) = 4(a+r) = 6(t+r) = 1$, so $a+t = \frac{1}{3}$, $a+r = \frac{1}{4}$, and $t+r = \frac{1}{6}$. Adding these up and dividing by 2 gives $a+r+t = \frac{3}{8}$, so it will take $\frac{1}{a+r+t} = \boxed{\frac{8}{3}}$ days for them to pick all the apples.

49. Let us consider $n \mod 3$. When $n \equiv 0 \mod 3$, $n^2 + 2n + 3 \equiv 0 + 0 + 3 \equiv 0 \mod 3$. When $n \equiv 1 \mod 3$, $n^2 + 2n + 3 \equiv 1 + 2 + 3 \equiv 0 \mod 3$. Finally, when $n \equiv 2 \mod 3$, $n^2 + 2n + 3 \equiv 4 + 4 + 3 \equiv 2 \mod 3$. Thus, $3 | n^2 + 2n + 1$ exactly when n is congruent to 0 or 1 mod 3. There are 670 numbers that are $0 \mod 3$, namely $3 \cdot 1, 3 \cdot 2, \ldots, 3 \cdot 670 = 2010$. There are 671 numbers that are $1 \mod 3$, namely $3 \cdot 0 + 1, 3 \cdot 1 + 1, \ldots, 3 \cdot 670 + 1 = 2011$. Thus there are $670 + 671 = \boxed{1341}$ numbers that satisfy the restrictions.

50. The area of a polygon with 20,000 sides can be approximated by the area of a circle. The polygon has a diagonal of length 20, which corresponds to a diameter of 20 in the approximating circle. A circle with a diameter of 20 has a radius of 10 for an area of $\pi \cdot 10^2 = 100\pi$, which is approximately $\boxed{314}$.