1 Grade 6 Solutions

1. At the end, Sue is on rung number $9+6-2+3-9+11 = 18$. Since this is also the top rung, there must be $\boxed{18}$ rungs in total.

2. The answer is $\boxed{2}$.

3. In the worst case scenario, Chef J grabs 2 chips and 2 truffles. However, whichever chocolate he picks next, there are going to be at least 3 pieces of the same type of chocolate. Therefore, the answer is $\boxed{5}$.

4. $1.354 + 0.79 + 2.005 + 1.8 + 4.05 + 0.001 = \boxed{10}$.

5. There are $\boxed{9}$ such numbers. We can find them simply by listing them all out: 2011, 2101, 2110, 1201, 1210, 1120, 1102, 1012, and 1021.

6. $11^4 = 11^2 \cdot 11 \cdot 11 = 121 \cdot 11 \cdot 11 = 1331 \cdot 11 = \boxed{14641}$.

7. Since x^2y is constant for all values of x and y, we have that x^2y is always equal to $6^2 \cdot 4 = 144$. Therefore, we need y such that $4^2 \cdot y = 144$, so $y = \boxed{9}$.

8. Notice that the sum of the integers from $-n$ to n is exactly 0. We can verify this by grouping each number with its opposite and noticing that each pair adds up to 0. Thus, the sum of the numbers from -15 to 15 is 0. If we include the number 16 into our sum, we find that the sum of the numbers from -15 to 15 is 16. There are 32 consecutive integers between -15 and 16 inclusive, so the answer is $\boxed{32}$.

9. There are 4 choices for the first person, 3 for the second, 2 for the third, and 1 for the fourth. Thus, there are $4 \cdot 3 \cdot 2 \cdot 1 = \boxed{24}$ ways to line up the people.

10. Each cycle of the pattern consists of 3 forward steps and then 1 back step, which is a net gain of 2 forward steps. After 99 cycles, or 99 backward steps, the baby chick has made a net of $99 \cdot 2 = 198$ steps, making it 2 steps away from the traffic light. Therefore, the next step forward will bring it to its destination, so it will have made a total of $\boxed{99}$ backward steps.

11. We shall use ratios. $48 \text{ frogs} \cdot \dfrac{5 \text{ frigs}}{6 \text{ frogs}} \cdot \dfrac{3 \text{ fregs}}{4 \text{ frigs}} \cdot \dfrac{1 \text{ frag}}{2 \text{ fregs}} = \boxed{15}$ frags.

12. Note that we can rearrange the expression to $(2-1)+(4-3)+(6-5)+\cdots+(18-17)+(20-19) = \underbrace{1+1+\cdots+1}_{10} = \boxed{10}$.

13. The probability the first word is "cake" is $\frac{1}{4}$, the probability that the second word is "is" is $\frac{1}{4}$, and the probability that the third word is "cake" is $\frac{1}{4}$. Since these probabilities are independent of one another, the final probability is $\frac{1}{4} \cdot \frac{1}{4} \cdot \frac{1}{4} = \boxed{\frac{1}{64}}$.

14. The probability that Rohil selects an odd number from 1 to 5 inclusive is $\frac{3}{5}$, and the probability that Michael selects a prime number from 1 to 6 inclusive is $\frac{3}{6} = \frac{1}{2}$. Hence, the answer is $\frac{3}{5} \cdot \frac{1}{2} = \boxed{\frac{3}{10}}$.

15. $g(1) = 2$, $f(g(1)) = f(2) = 3$, $g(f(g(1)) = g(3) = 6$, $f(g(f(g(1)))) = 7$, $g(f(g(f(g(1))))) = 14$, so $f(g(f(g(f(g(1)))) = \boxed{15}$.

16. A triangle with side lengths of 6, 8, and 10 is a right triangle with legs of lengths 6 and 8 and a hypotenuse of length 10 because the triangle satisfies the Pythagorean Theorem ($6^2 + 8^2 = 10^2$.) The area is therefore half the product of the legs: $\frac{1}{2} \times 6 \times 8 = \boxed{24}$.

17. It can be easily verified that the statement holds true when $n = 3, 4,$ or 5. For $n > 5$, it can easily be seen that the longest diagonal is longer than the shortest. Thus, our answers are $\boxed{3, 4, \text{ and } 5}$.

18. There are no such palindromes. Suppose, for the sake of contradiction there exists a palindrome with 8 digits and whose digits sum to 15 be $abcddcba$. Then we have that $2(a + b + c + d) = 15$, which is impossible since 15 is even, so our answer is $\boxed{0}$.

19. The sums of the angles of $\triangle ADE$ and $\triangle ABC$ sum to 180, so $\angle B + \angle C = 180° - \angle A = \angle D + \angle E = 120°$, so $\angle C = 120° - 100° = \boxed{20°}$.

20. We know that all squares of real numbers are nonnegative. Thus, if two squares add to 0, both of them must be zero. We must therefore have $2x+3y = 5$ and $x-2y = -7$. Tripling the second equation and adding it to twice the first equation gives us $7x = -11 \implies x = \boxed{-\frac{11}{7}}$.

21. Let CD be the diameter of the semicircle, and let M be the midpoint of CD. The area of the semicircle is $\frac{2^2\pi}{2} = 2\pi$, and the area of the triangle is $\frac{4MB}{2} = 2MB$. Since the area of the total figure is $2\pi+2MB = 16$, we have that $MB = 8$. Thus, $AB = AM+MB = 2+8 = \boxed{10}$.

22. We have $7a9 + 2b4 = cd13$. Since there is a carry in the ones digit, we must have either $a+b+1 = 1$ or $a+b+1 = 11$. In the former case, $7a9+2b4 < 1000$, which is impossible since $cd13$ is a four-digit number. Thus, $a + b = 11$, whence c and d must be 1 and 0, respectivelly, so our answer is $10 + 1 + 0 = \boxed{11}$.

23. There are 7! ways to arrange 7 distinct letters. However there are three A's in the word. If we treat these as distinct letters we have the above number. However because we cannot distinguish between the A's, we need to divide by the number of times we counted the A's as if they were distinct, which is the same as the number of ways we can arrange the A's, which is 3!. There are therefore $\frac{7!}{3!} = \boxed{840}$ arrangements of letters in the word ANAGRAM.

24. Note that $49^{27x} = (7^2)^{27x} = 7^{54x}$. As a result, $\frac{49^{27x}}{7^{9x}} = 7^{45x}$. Since this is equal to $49 = 7^2$, we have that $x = \boxed{\frac{2}{45}}$.

25. If $2x + 5y = 100$, since $2x$ and 100 are both even, $5y$ must be even too, so y must be even. Since $x \geq 6$, $5y \leq 100 - 2x \leq 88$, so $y \leq 17$. From here, we can list all the solutions: $(x, y) = (35, 6), (30, 8), (25, 10), (20, 12), (15, 14), (10, 16)$. There are $\boxed{6}$ solutions in this list.

26. Rewrite the equation using the difference of squares: $(a+b)(a-b) = 36$. Notice that this has an integer solution to it if $(a+b)+(a-b) = 2a$ is even. Thus, the ordered pair $(a+b, a-b)$ has five solutions: $(6, 6), (-6, -6), (2, 18), (18, 2), (-2, -18), (-18, -2)$, each of which corresponds to exactly one ordered pair (a, b). Thus, there are $\boxed{6}$ solutions.

27. By inspection, $x = -1$ satisfies the equation. Factoring, we find that $x^3 + x^2 + x + 1 = (x+1)(x^2+1)$. $x^2 + 1$ has no real roots, so the only real x that satisfies the equation is $x = \boxed{-1}$.

28. After a cycle of one day and one night, the snail has a net movement of 2 meters up the tube. After the ninth day, the snail is $8 \cdot 2 + 3 = 19$ meters from the bottom of the tube. During the ninth night, the snail sinks 1 meter down, and reaches the top of the 20 meter tube on the tenth day. Therefore, the answer is $\boxed{10}$.

29. It is easy to see that the number n appears between positions $(1 + 2 + \cdots + (n-1)) + 1$ and $(1+2+\cdots+n)$ inclusive, that is, $\frac{n(n-1)}{2} + 1$ and $\frac{n(n+1)}{2}$ inclusive. Since 200 lies between $190 = \frac{20 \cdot 19}{2} + 1$ and $210 = \frac{20 \cdot 21}{2}$ inclusive, our answer is $\boxed{20}$.

30. If n^2 is a cube, n must also be a cube; if n^3 is a square, then n must also be a square. Thus, n must be both a square and a cube, so it must be a sixth power. The smallest sixth power larger than 1 is $2^6 = \boxed{64}$.

31. In the binomial expansion of $\left(x + \frac{1}{x}\right)^{2011}$, all the terms are of the form $cx^m \left(\frac{1}{x}\right)^n$, with $m + n = 2011$. In order to attain the term x^2, we must have $m - n = 2$. Solving these equations gives $(m, n) = (\frac{2013}{2}, \frac{2009}{2})$. But since m and n must be integers, the x^2 term must have coefficient $\boxed{0}$.

32. The number 9^n ends with 1 if n is even, and ends with 9 if n is odd. Since $8^{7^{6^{5^{4^{3^{2^1}}}}}}$ is even, the last digit of $9^{8^{7^{6^{5^{4^{3^{2^1}}}}}}}$ is $\boxed{1}$.

33. The expression on the left hand side will be equal to 1 only when the exponent is equal to 0 or if $x + 1$ is equal to -1 or 1. The exponent will be equal to 0 when $x = -1$ or -2, and the base is equal to 1 when $x = -2$ or when $x = 0$. We verify that $x = 0$ and $x = -2$ satisfy the equation, while $x = -1$ yields an expression of the form 0^0, which is undefined, so the only x satisfying the equation are 0 and -2, so our answer is $\boxed{2}$.

34. After drawing the figure, we find that half the chord (which has a length of 6), the radius of the circle, and the line from O to AB form a right triangle with legs 3 and 6. Applying the Pythagorean Theorem, we find that the hypotenuse is $3\sqrt{5}$, which is also the radius. The area is therefore $\pi \cdot \text{radius}^2 = \pi \left(3\sqrt{5}\right)^2 = \boxed{45\pi}$.

35. Subtract the second equation from the first to get $3x^2 - 8x = 35 \implies 3x^2 - 8x - 35 = 0$. This factors as $(3x+7)(x-5) = 0 \implies x = 5$ or $x = -\frac{7}{3}$. However, only one of these is a solution to the first equation, namely $x = \boxed{5}$.

36. WLOG let the radius of the circle be 1. Then the total area of the circle is π. The length of the median of the equilateral triangle is $\frac{3}{2}$, so the side length is $\frac{\frac{3}{2}}{\frac{\sqrt{3}}{2}} = \sqrt{3}$. Thus the area of

the triangle is $\frac{1}{2} \cdot \frac{3}{2} \cdot \sqrt{3} = \frac{3\sqrt{3}}{4}$, and the probability that a point picked is inside the triangle is $\frac{3\sqrt{3}}{4\pi}$. The probability that the point is NOT inside the triangle is thus $\boxed{1 - \frac{3\sqrt{3}}{4\pi}}$.

37. Trial and error gives that $\boxed{7}$ is the smallest positive integer that cannot be represented as the sum of three not necessarily distinct perfect squares.

38. $((3!)!)! = (6!)! = 720!$; the number of zeroes at the end of 720 is $\lfloor \frac{720}{5} \rfloor + \lfloor \frac{720}{25} \rfloor + \lfloor \frac{720}{125} \rfloor + \lfloor \frac{720}{625} \rfloor = 144 + 28 + 5 + 1 = \boxed{178}$.

39. $1^2 - 2^2 + 3^2 - 4^2 + \cdots - 50^2 + 51^2 = 1 + (3+2)(3-2) + (5+4)(5-4) + \cdots + (51+50)(51-50)$. (Note: $a^2 - b^2 = (a+b)(a-b)$.) This is equal to $1 + 5 + 9 + \cdots + 101 = 13 \cdot 102 = \boxed{1326}$.

40. Let y be the length of the side parallel to the river, and let x be the lengths of the other two sides. We must have $2x + y = 200$. We seek to maximize the area, xy, that is enclosed by the fence and the river. Rearranging, we get $y = 200 - 2x$, so the area is $xy = x \cdot (200 - 2x) = 200x - 2x^2$, which is a quadratic in x. The vertex, or maximum (because the coefficient of x^2 is negative), occurs at $x = \frac{-200}{(2 \cdot (-2))} = 50$. When $x = 50, y = 200 - 2 \cdot 50 = 100$. The area is then $xy = 50 \cdot 100 = \boxed{5000}$.

41. Let $x = BD$ and $y = DC$. Clearly, $x + y = 5$. Also, by the angle bisector theorem, $\frac{x}{y} = \frac{4}{6} = \frac{2}{3}$, so $3x = 2y = 2(5 - x)$, so $5x = 10$, so $x = BD = \boxed{2}$.

42. Call the roots of $x^2 + 4x - 2 = 0$ a and b. From Vieta's formulas, we know that $ab = -2$ and $a + b = -4$. Write the desired quadratic as $x^2 - qx + r$, for some q and r. Again by Vieta's forumlas, we have that $r = (ab)^2$ and $q = a^2 + b^2$. r is simply $(-2)^2 = 4$. To find q, we note that $(a+b)^2 = a^2 + 2ab + b^2$. Rearranging, we have that $a^2 + b^2 = (a+b)^2 - 2ab = (-4)^2 - 2(-2) = 20$. The quadratic equation we seek is therefore $\boxed{x^2 - 20x + 4 = 0}$.

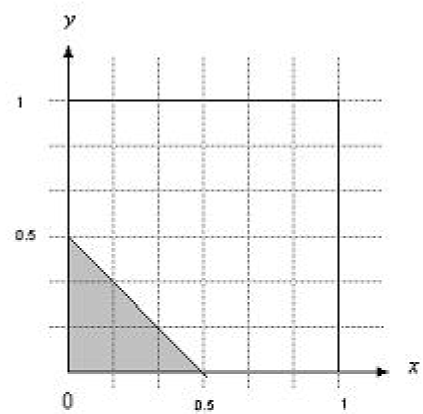
43. Note that the line segments A_1B_1, B_1C_1, C_1A_1 divides triangle ABC into four identical smaller triangles, and similarly A_2B_2, B_2C_2, C_2A_2 divides triangle $A_1B_1C_1$ into four even smaller identical triangles. Therefore, the area of triangle ABC is four times that of triangle $A_1B_1C_1$, which in turn has area four times that of $A_2B_2C_2$. Hence, the ratio of the area of triangle $A_2B_2C_2$ to the area of triangle ABC is $\boxed{\frac{1}{16}}$.

44. Since Archimedes' position in the line is fixed, it is enough to find the number of ways to arrange Bernoulli, Cauchy, Descartes, and Euler in a line. Because Descartes and Euler must stand next to each other, we temporarily view them as a single unit. The number of ways to arrange Bernoulli, Cauchy, and the "Descartes-Euler unit" is $3 \times 2 \times 1 = 6$, and the number of ways to arrange Descartes and Euler in the "unit" is 2, so our answer is $6 \times 2 = \boxed{12}$.

45. The ratio of the areas of two similar polygons is the square of the ratio of similitude between the two polygons, so we need to find $\left(\frac{B_1B_2}{A_1A_2}\right)^2$. If we let M be the midpoint of B_1B_2, we

see that A_2B_2M is a 30-60-90 triangle, so $\dfrac{B_2B_1}{A_3A_2} = \dfrac{B_2M}{B_3A_2} = \dfrac{\sqrt{3}}{2}$, so the desired ratio is $\left(\dfrac{\sqrt{3}}{2}\right)^2 = \boxed{\dfrac{3}{4}}$.

46. Consider the attached diagram. If we choose x, y randomly in this 1×1 square, the probability that their sum is less than $\dfrac{1}{2}$ is the probability that (x, y) lies in the shaded region. The area of the shaded region is $\dfrac{1}{8}$, while the area of the whole region is 1, so our answer is $\boxed{\dfrac{1}{8}}$.



47. In general, $(x+y)^2 + (x-y)^2 = 2(x^2+y^2)$. Thus, $2 \cdot 929 = (23-20)^2 + (23+20)^2 = 46^2 + 3^2$, so our answer must be $43 + 3 = \boxed{46}$. (It can be verified that this decomposition is unique.)

48. If $x - \dfrac{1}{x} = 3$, then $\left(x - \dfrac{1}{x}\right)^2 = 9$, or $x^2 + \dfrac{1}{x^2} - 2 = 9$, so $x^2 + \dfrac{1}{x^2} = \boxed{11}$.

49. First, notice that $\dfrac{x^5-1}{x-1} = x^4+x^3+x^2+x+1$. Letting $x = 10^{20}$ gives us that the subtrahend is $(10^{80}+10^{60}+10^{40}+10^{20}+1)$. Next, we realize that $10^{100}-1 = 999\ldots999$, or a string of 100 9's, which has a sum of 900. Since we are subtracting the number $(10^{80}+10^{60}+10^{40}+10^{20}+1)$, which is composed of a total of 5 1's and many 0's, and since no borrowing is done, the answer is just $900 - 5 = \boxed{895}$.

50. The area of a polygon with 20,000 sides can be approximated by the area of a circle. The polygon has a diagonal of length 20, which corresponds to a diameter of 20 in the approximating circle. A circle with a diameter of 20 has a radius of 10 for an area of $\pi \cdot 10^2 = 100\pi$, which is approximately $\boxed{314}$.