1. Note that we can fold an octagon symmetrically in two ways—either through the midpoints of two opposite sides or through two opposite vertices. Each has four ways, since there are four pairs of opposite sides and four pairs of opposite vertices. Thus the answer is $4+4 = \boxed{8}$.

2. $164 \div 4 = \boxed{41}$.

3. Change the fractions to have common denominator $3 \cdot 7 = 21$. The sum is thus equal to $\frac{9}{21} + \frac{49}{21} = \boxed{\frac{58}{21}}$.

4. Since each chair has 4 legs, there are $6 \cdot 4 = 24$ legs from the chairs. Since each table has 3 legs, there are $4 \cdot 3 = 12$ legs from the tables. Thus, there are a total of $24 + 12 = \boxed{36}$ legs.

5. Each second, Tom will run 25 meters closer to Jerry. This gives us the equation $25 \cdot t = 100$, where t denotes the time it takes to catch Jerry. So the answer is $t = \frac{100 \text{ meters}}{25 \text{ meters/second}} \cdot \frac{1 \text{ minute}}{60 \text{ seconds}} = \boxed{\frac{1}{15} \text{ minute}}$.

6. Since 1126 is even, one of the primes is 2. Since we are told that the number is the product of exactly two primes, we know that the other prime is $\frac{1126}{2} = 563$ without any computation. Therefore, the sum is $563 + 2 = \boxed{565}$.

7. The average is equal to the sum of the test scores divided by the number of tests. Thus Licheng's average is $\frac{60 + 95 + 94}{3} = \boxed{83}$.

8. We can simply multiply it out, or we can look for a pattern: $11^2 = 121$, $111^2 = 12321$, etc. So we can deduce that $111111^2 = \boxed{12345654321}$.

9. Kevin and Chester place higher than Dominic, so neither Kevin now Chester can be 4^{th} place. Ryan does not finish in the bottom 2, so he cannot be 4^{th} place either. Therefore, Dominic must be in 4^{th} place.

10. Adding the two equations together, we arrive at $3x + y + x - y = 4x = 56$, and so $x = \boxed{14}$.

11. Since each person must take at least one language, if 170 people take Arabic then $240 - 170 = 70$ students must take Just Swahili. Since there are a total of 120 students taking Swahili, and 70 of them only take Swahili, $120 - 70 = \boxed{50}$ students must take both Swahili and Arabic.

12. Solution 1: Let x be the middle integer. We have that $(x - 8) + (x - 7) + \cdots + (x + 7) + (x + 8) = 0$. This simplifies to $x + (x + 1) + (x - 1) + \cdots + (x + 8) + (x - 8) = 17x = 0$, meaning that $x = 0$. So the smallest of the consecutive integers is $x - 8 = \boxed{-8}$.

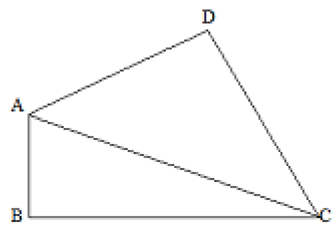
    Solution 2: Since 17 consecutive integers sum to 0, some of them must be negative and some must be positive, so 0 must be one of the numbers included. In fact, since they are consecutive integers that sum to 0, there must be an equal number of positive and negative numbers, or $\frac{(17 - 1)}{2} = 8$ numbers each. Therefore, the smallest of these 17 consecutive integers is the 8th negative integer, or $\boxed{-8}$.

13. It is given that $2a = 3b$ and $7b = 5p$, where a is the weight of apples, b is the weight of bananas and p is the weight of pears. Therefore we have that $90b + 90p = 90b + 90\left(\frac{7b}{5}\right) = 126b = 126\left(\frac{2a}{3}\right) = 84a$. Therefore our answer is $\boxed{84}$.

14. Let a, b denote the length and width of the rectangle. We know that $2a + 2b = 30$, and hence $a + b = 15$. We can see that as the rectangle becomes more square-like, the area increases, i.e. the two sides must be as close as possible. Since both sides are integers, the area is maximized when $a = 7$ and $b = 8$, which yields an area of $7 \cdot 8 = \boxed{56}$.

15. Every hour, Bill the Builder lays 100 bricks, while his dog knocks down $10\,(2) = 20$. Thus every hour $100 - 20 = 80$ bricks are laid, and so Bill the Builder will lay all 960 bricks in $\frac{960}{80} = \boxed{12}$ hours.

16. Let the two numbers be $3a$ and $3b$, since they both share a common factor of 3. We have $3a + 3b = 21$, or $a + b = 7$. Since a and b are positive, there are three choices for the numbers. For the GCF to be 36, both $3a$ and $3b$ must be divisors of 36, and the only such case that works when $a + b = 7$ is when $a = 3$ and $b = 4$ (or vice-versa). Therefore the final answer is equal to $(3 \cdot 3) \cdot (4 \cdot 3) = \boxed{108}$.



17. In the diagram above, we can see that by drawing segment $\overline{AC}$, we get a hypotenuse of both right triangles ABC and CDA. To confirm this, we note that $AB^2 + BC^2 = 7^2 + 24^2 = CD^2 + AD^2 = 15^2 + 20^2 = 25^2$. Therefore, $AC = 25$ and we can find the area of the quadrilateral by adding the areas of the two triangles, which is $\frac{1}{2}\,(7)\,(24) + \frac{1}{2}\,(20)\,(15) = 84 + 150 = \boxed{234}$.

18. The prime factorization of 2013 is $3 \cdot 11 \cdot 61$ (to do this quickly, it helps to remember the divisibility rules for 3 and 11). Each factor of 2013 can be written in the form $3^a \cdot 11^b \cdot 61^{\mathrm{c}}$, where the choices for a, b, and c are either 0 or 1. Therefore, there are $2 \cdot 2 \cdot 2 = \boxed{8}$ factors

19. We know, from 1 across and 1 down, that a two-digit perfect cube and a two-digit perfect square must have the same tens digit. The only options for the tens digit are therefore 2 (27 and 25) or 6 (64 and 64). However, if the tens digit is 6, then by 3 across we must have a multiple of 13 that begins with a 4. However, no such multiple exists, so the tens digit cannot be 6. Instead, it must be 2. We are now looking for a two-digit multiple of 13 that begins with a 5, and a two-digit multiple of 8 that begins with 7. We see that putting 2 in the last two boxes, giving 72 and 52, satisfies the criteria. Our final crossword looks like this:

2	7
5	2

20. See #19

21. See #19

22. See #19

23. Work backwards. Notice that when $b = 1$, $a\#b$ simplifies to $a - a = 0$. Since we are being asked to find (something)#1, this is just $\boxed{0}$.

24. First, we know that any even number of stamps can be sorted in groups of 2. Then for every odd number greater than or equal to 5, we can sort them by using one group of 5 and then sort the remaining into groups of 2. Therefore, the biggest number of stamps that cannot be sorted is the largest odd number less than 5, or $\boxed{3}$.

25. Using the formula $1+2+3+\cdots+n = \dfrac{n(n+1)}{2}$, we see that $3+4+\cdots+n = 1+2+\cdots+n-(1+2) = \dfrac{n(n+1)}{2} - 3 = 1323$, which simplifies to $n(n+1) = 2652$, which gives $n = \boxed{51}$.

26. On the n-th day, Steven meets n more people, so he has met $1+2+3+\cdots+n = \dfrac{n(n+1)}{2}$ people in total. Since n and $n+1$ have different parities (meaning that one is even and one is odd), either $\dfrac{n}{2}$ or $\dfrac{n+1}{2}$ must be a multiple of 32. Thus, the smallest n is when $\dfrac{n+1}{2}$ is the smallest multiple of 32. This happens when $\dfrac{n+1}{2} = 32$, or $n = \boxed{63}$.

27. We want to maximize a^b and minimize cd. To maximize a^b we need to make it as positive as possible. This happens when we let $a = 3$ (or -3) and $b = 2$. Next, to minimize cd, we need to make it as negative as possible, so we want to choose the largest combination of a positive number and a negative number. Since we are already choosing one 3 (for a), the most negative product possible is -6, which happens when $c = 3$ and $b = -2$. Consequently, we get that $a = -3$, and so our final answer is $(-3)^2 - (3)(-2) = \boxed{15}$.

28. Let x be the number of Doduos and y be the number of Dodrios. We get the system of equations $2x + 3y = 291$and $2x + 2y = 222$. Subtracting the second equation from the first gives us $y = 69$. Plugging this into the second equation, we get $x = \boxed{42}$.

29. We are looking for numbers not divisible by 2 or 3, since these are the only prime factors of 48. There are $\dfrac{48}{2} = 24$ multiples of 2 and $\dfrac{48}{3} = 16$ multiples of 3 that are at most 48, giving us a sum of 40 multiples of 2 or 3. However, we over-counted multiples of 6 twice, since they are both divisible by 2 and 3, so we must subtract the number of multiples of 6 from our count, which is $\dfrac{48}{6} = 8$. Hence there are $40 - 8 = 32$ multiples of 2 or 3 that are at most 48, and therefore there are $48 - 32 = \boxed{16}$ numbers that are relatively prime to 48.

30. Let the original fraction be $\dfrac{a}{b}$. Thus we have $\dfrac{a+27}{b+36} = \dfrac{a}{b}$. Cross-multiplying, we get $ab+27b = ab + 36a \rightarrow 27b = 36a$. Thus it follows that, in simplest form, $\dfrac{a}{b} = \dfrac{27}{36} = \boxed{\dfrac{3}{4}}$.

31. Let her number be equal to $100a + 10b + c$, where a, b, c are the digits of the number (i.e. they are integers satisfying $0 < a < 10$ and $0 = b, c < 10$). We have that $a^2 + b^2 + c^2 = 2(a+b+c) = 2a + 2b + 2c$. We can move all terms to one side and factor to get $a^2 - 2a + 1 + b^2 - 2b + 1 + c^2 - 2c + 1 = (a-1)^2 + (b-1)^2 + (c-1)^2 = 3$. Since a, b, c are integers, the only solution to this equation is $a = b = c = 2$. Therefore her number is $\boxed{222}$.

32. For the first few numbers, we can simply evaluate n^n. For example, $1^1 = 1,\ 2^2 = 4,\ 3^3 = 27$ and $4^4 = 256$. We see that only 1 satisfies the condition out of these numbers.

For 5^5, we note that, since $5 \cdot 5$ ends in a 5, any power of 5 will end in 5. We can see that 6 works similarly as well.

For 7^7, we see that the ones digit cycles with increasing powers as such: $7, 9, 3, 1, 7, 9, 3, 1, \ldots$. Thus 7^7 has a ones digit of 3, and so 7 does not work.

For 8^8, we see that the ones digit also cycles: $8, 4, 2, 6, 8, 4, 2, 6, \ldots$. So 8^8 has a ones digit of 6 and therefore does not work.

Lastly, 9^9 ends with a 9, as the ones digit alternate between 9 and 1. So there are 4 numbers that work—1, 5, 6, and 9. Therefore, the probability is $\boxed{\dfrac{4}{9}}$.

33. Since line segment BEC is a diagonal of square $ABCD$, we have that $< ECG = 45°$. Hence right triangle CEG is isosceles. Therefore, since $EFDG$ we get that $CD = CG + GD = EG + EF = 3 * EG = 12$. Solving, we get EG= 4. And so the area of triangle AEC is equal to $\frac{1}{2}$∗base∗height= $\frac{1}{2} * AC * CG = \frac{1}{2} * 12 * EG = 6 * 4 = \boxed{24}$.

34. For a positive integer of the form $p_1^{e_1} \ldots p_n^{e_n}$, the number of factors it has is $(e_1 + 1) \ldots (e_n + 1)$. Therefore, for the number to have exactly 12 factors, the expression $(e_1 + 1) \ldots (e_n + 1)$ must equal 12. Prime factoring 12, we see that we can satisfy this condition in many ways:

Case 1: The number has only one factor, i.e. it is of the form p^{e_1}. Then, in order for it to have 12 factors, we must have $e_1 + 1 = 12 \rightarrow e_1 = 11$. Therefore, it must be of the form p^{11}. The smallest prime p is 2, so the smallest number for this case is $2^{11} = 2048$.

Case 2: The number has two factors, i.e. it is of the form $p_1^{e_1} p_2^{e_2}$. Then, for this to have 12 factors, we must have $(e_1 + 1)(e_2 + 1) = 12$. This can be done in two ways: either $p_1^3 p_2^2$ or $p_1^5 p_2^1$. Either way, we will choose $p_1 = 2$ and $p_2 = 3$ because those are the two smallest primes, and we want the smallest prime possible for the largest exponent. These two give $8 \cdot 9 = 72$ and $32 \cdot 3 = 96$, respectively.

Case 3: The number has three factors, i.e. it is of the form $p_1^{e_1} p_2^{e_2} p_3^{e_3}$. Then, for this to have 12 factors, we must have $(e_1 + 1)(e_2 + 1)(e_3 + 1) = 12$. This can only be done in one way: if $e_1 = 2$, $e_2 = 1$, and $e_3 = 1$ (letting any $e_n = 0$ reverts to one of the earlier cases). Therefore, the number must be of the form $p_1^2 p_2^1 p_3^1$. Following the rule of giving the largest exponent the smallest possible prime, we get the smallest possible value of that expression by setting $p_1 = 2$, $p_2 = 3$, and $p_3 = 5$, giving us a final answer of $4 \cdot 3 \cdot 5 = 60$.

Of the three cases, the smallest possible number with exactly 12 factors is $\boxed{60}$.

35. Let us first count the maximum number of intersection points between 10 circles. Suppose we have n circles with maximum number of intersections, and we are trying to add another one. Then, the last circle can intersect each of n circles at maximum of 2 points, implying that the last circle can add a maximum of $2n$ intersection points. Thus, when we have $n + 1$ circles, the maximum number of intersections is $2 + 4 + \cdots + 2n = n(n + 1)$. Evaluating this for $n = 9$, we find that the maximum number of intersections for 10 circles is 90. Now, we account for the two lines. Again, each of two lines can intersect each of 10 circles at 2 points, so that adds $2 \cdot 2 \cdot 10 = 40$ intersections. Finally, because two lines can intersect maximum of once, the answer is $90 + 40 + 1 = \boxed{131}$.

36. Using the formula for an infinite geometric series, $\frac{1}{a}+\frac{1}{a^2}+\frac{1}{a^3}+\cdots=\frac{\frac{1}{a}}{1-\frac{1}{a}}$. Multiplying the top and bottom by a, we have that $\frac{1}{a-1}=\frac{4}{5}$. We can solve for a to get $a=\frac{9}{4}$. Let $S=\frac{1}{a^2}+\frac{1}{a^4}+\frac{1}{a^6}+\cdots$. Then $aS=\frac{1}{a}+\frac{1}{a^3}+\frac{1}{a^5}+\cdots$. Therefore, $aS+S=\frac{1}{a}+\frac{1}{a^2}+\frac{1}{a^3}+\cdots=\frac{4}{5}$. Plugging in $a=\frac{9}{4}$, we have that $\frac{9}{4}\cdot S+S=\frac{13}{4}\cdot S=\frac{4}{5}$. Solving for S, we get $S=\boxed{\frac{16}{65}}$.

37. By the Angle Bisector Theorem, $\frac{5}{BD}=\frac{7}{6-BD}$. Cross multiplying and solving gives us $BD=\boxed{\frac{5}{2}}$.

38. A fraction, when written out in decimal form, will terminate if it can be written as a fraction in the form $\frac{x}{10^y}y(x$ and y are positive integers). Therefore, it will not terminate if its denominator has prime factors other than 2 or 5 (the only prime factors of 10). Therefore, we are looking for the number of numbers between 1 and 50 that are not divisible by 2 or 5. There are $\frac{50}{2}=25$ numbers divisible by 2 and $\frac{50}{5}=10$ numbers divisible by 5, giving us a total of $25+10=35$ numbers. However, we have counted the $\frac{50}{10}=5$ multiples of 10 twice. Therefore, there are $35-\frac{50}{10}=30$ numbers that are multiples of 2 or 5, and so $50-30=20$ numbers that are not divisible by 2 or 5. Lastly, since $\frac{1}{1}$ terminates, we have to subtract 1 from our answer, giving a final answer of $\boxed{19}$.

39. Let the numbers be $2k-4,\ 2k-2,\ 2k,\ 2k+2,\ 2k+4$. Multiplying all numbers gives us $(2k-4)(2k+4)(2k-2)(2k+2)(2k)=32(k-2)(k+2)(k-1)(k+1)(k)$. Thus 32 must divide this number. Furthermore, there must be at least one integer out of the five consecutive integers $k-2, k-1, k, k+1, k+2$ that is divisible by 2, at least one divisible by 3, at least one divisible by 4 and at least one divisible by 5 (for example, take 1, 2, 3, 4, 5). Therefore the largest number that must always be a factor of this is $2^5\cdot 2\cdot 3\cdot 4\cdot 5=\boxed{3840}$.

We can see that this it cannot be any larger since $2\cdot 4\cdot 6\cdot 8\cdot 10=3840$.)

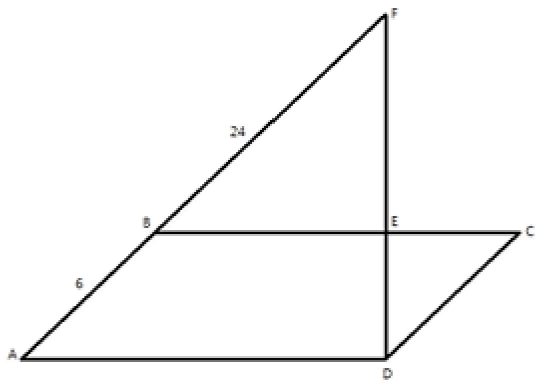
40. First, square $a+b=5$ to get $(a+b)^2=a^2+2ab+b^2=25$. Then, we can subtract $2ab=14$ from this equation to get $a^2+b^2=11$. Now we square a^2+b^2 to get $\left(a^2+b^2\right)^2=a^4+2a^2b^2+b^4=121$. However, since $2a^2b^2$ is just $2(ab)^2=98$, we can subtract that from the equation to get $a^4+b^4=121-98=\boxed{23}$.

41. If a number is congruent to 1 mod 3 and 1 mod 5, then it must be congruent to 1 mod 15. We can list the first few numbers that leave a remainder of 1 when divided by 15: $1, 16, 31, 46, 61, 76, 91, 106, ?$. We see that $\boxed{91}$ is the first number that is divisible by 7.

42. Using the formula for the number of factors, we see that the only "pretty good" numbers are in the form p^{q-1} for some primes p, q (since its factors will be $1, p, p^2, \ldots, p^{q-1}$). If $p=2$, we can have $q=2,\ 3,\ 5$ or 7. If $p=3$, $q=2,\ 3$, or 5. If $p=5$ or 7, $q=2$ or 3. Finally, for all primes p greater than 7, q can only be 2. Since we can count 21 primes greater than 7 (11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97), our final answer is $1*4+1*3+2*2+21*1=\boxed{32}$.

43. We want to find $3^{2013} \pmod{16}$. We can note that $3^4 = 81 \equiv 1 \bmod 16$, and thus we can write this as $\left(3^4\right)^{503} \cdot 3 \equiv 1^{503} \cdot 3 \equiv \boxed{3} \bmod 16$.

44. Let $P(x)$ be the probability that James gets x heads. We want James to get at least 4 heads. Note that $P(h) = P(7-h)$, since the probability of flipping h heads is equal the probability of flipping h tails. Thus we see that $1 = P(0) + P(1) + \cdots + P(7) = 2\left[P(4) + \cdots + P(7)\right]$. Solving, we see that the probability of getting at least 4 heads is equal to $\boxed{\frac{1}{2}}$.



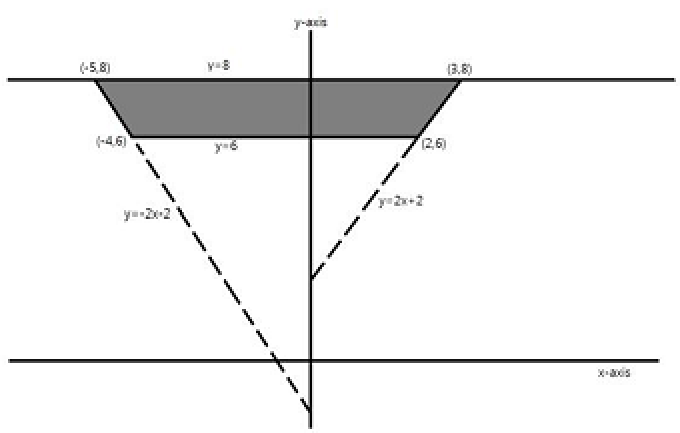
45. Using parallel lines $\overline{AF}$ and $\overline{CD}$, it can be seen that $\triangle CDE$ is similar to $\triangle BFE$ by AAA Similarity. Note that since $ABCD$ is a parallelogram, $CD = AB = 6$, and we are given that $BF = 24$. Therefore, the ratio of the sides of these two triangles is $24 : 6 = 4 : 1$. The ratio of the areas is the ratio of the sides squared, or $4^2 = \boxed{16}$.

46. There are three cases:

If $x < -4$, then both $x+4$ and $x-2$ are negative. So the inequality is $8 > y > -(x+4) - (x-2) = -2x-2$. Thus solving for $x$gives $x > -5$. Thus for $-5 < x < -4$, the function is $y > -2x-2$.

If $-4 \leq x \leq 2$, $x+4$ is positive but $x-2$ is negative. Therefore the inequality is $8 > y > (x+4) - (x-2) = 6$.

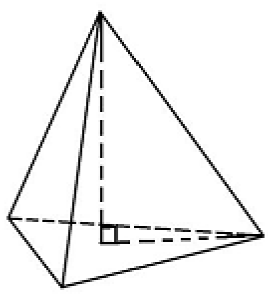
If $x > 2$, both terms in the absolute value are positive. So the inequality is $8 > y > (x+4) + (x-2) = 2x+2$. Solving for $x$gives $x < 3$. Thus for $2 < x < 3$, the function is $y > 2x+2$.



Plotting these piecewise functions, we see that the bounded region is a trapezoid with base lengths 8 and 6 and a height of 2. Therefore the area is $\frac{6+8}{2} * 2 = \boxed{14}$.

47. The triangular faces contribute $3 \cdot 8 = 24$ edges. The octagonal faces contribute $8 \cdot 6 = 48$ edges. However, each edge is counted twice since every edge is the meeting of two faces. Therefore, there are $\frac{24+48}{2} = \boxed{36}$ edges in total.

48. Let the two numbers be $2k+1$ and $2k-1$. Multiplying gives $(2k+1)(2k-1) = 4k^2 - 1 = 28 + n^2$, for some integer n. Simplifying, we get $(2k)^2 - n^2 = 29$. We then factor this further to get $(2k+n)(2k-n) = 29$. Since both k and n are integers and 29 is prime, the only positive solution to this equation occurs when $2k+n = 29$ and $2k-n = 1$. Solving, we get $k = \frac{15}{2}$, and so the smaller of the two consecutive even numbers is $2k - 1 = 2\left(\frac{15}{2}\right) - 1 = \boxed{14}$. (And we see that $14 * 16 = 224 = 28 + 14^2$)



49. Let s be the side of the tetrahedron. We know that the area of the base, an equilateral triangle, is $\frac{s^2\sqrt{3}}{4}$. Next, to find the height of the tetrahedron, we look at the right triangle inside that has a side of a triangle as the hypotenuse. The shortest side of this triangle is two thirds of the height of the triangle, and so by the Pythagorean Theorem we see that the height is equal to $\sqrt{s^2 - \left[\left(\frac{2}{3}\right)\left(\frac{s\sqrt{3}}{2}\right)\right]^2} = \sqrt{\frac{2}{3}s^2} = \frac{s\sqrt{2}}{\sqrt{3}}$. Finally, using the fact that the volume of a tetrahedron is $\frac{1}{3}Bh$ where B is the area of the base and h is the height length, we get that the volume is equal to $\frac{1}{3}\left(\frac{s^2\sqrt{3}}{4}\right)\left(\frac{s\sqrt{2}}{\sqrt{3}}\right) = \frac{\sqrt{2}}{12}s^3$. If $s = \sqrt{2}$, the volume is equal to $\frac{\sqrt{2}}{12}\sqrt{2}^3 = \frac{1}{3}$. Finally, since the volume of the cube is equal to the volume of the tetrahedron. We have that the edge length of the cube is $\sqrt[3]{\frac{1}{3}} = \boxed{\frac{1}{\sqrt[3]{3}}}$.

50. Since the sum of the angles in a quadrilateral is 360°, we can use this fact to find that $\angle D = 360 - 150 - 60 - 60 = 90^\circ$, or a right angle. Now we extend $\overline{AD}$ and $\overline{BC}$ to meet at point E, forming an equilateral triangle ($\angle E = 180 - 60 - 60 = 60^\circ$). Since the angle bisector and perpendicular bisector in an equilateral triangle are the same, E, X, and Y are collinear. We notice that since $\angle EAX = 60^\circ$ and $\angle EXA = 90^\circ$, $\triangle AEX$ is a 30-60-90 right triangle, and $EX = AX\sqrt{3} = 4\sqrt{3}$. Now we only need to find the value of $\overline{EY}$, which is the hypotenuse of $\triangle DEY$. Since $\angle DEY = 30^\circ$ and $\angle DEC = 60^\circ$, $\triangle DEY$ and $\triangle DEC$ are also

30-60-90 right triangles. From $\triangle DEC$, we know that $CD = 3$, so $DE = \dfrac{CD}{\sqrt{3}} = \sqrt{3}$. We then use this new piece of information for $\triangle DEY$, noting that $DY = \dfrac{DE}{\sqrt{3}}$ and $EY = 2DY$, meaning that $EY = \dfrac{2DE}{\sqrt{3}} = 2$. Therefore, the length of $\overline{XY}$ is $EX - EY = \boxed{4\sqrt{3} - 2}$.