

1. By counting, there are $\boxed{7}$ words in this question.
2. $1 + 4 + 2 + 8 + 16 + 32 = \boxed{63}$.
3. One pencil costs 25 cents, and we have 5 pencils, so the cost is $25 \cdot 5 = \boxed{125}$ cents.
4. A cube has $\boxed{12}$ edges.
5. $\frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} = \frac{30}{60} + \frac{20}{60} + \frac{15}{60} + \frac{12}{60} = \boxed{\frac{77}{60}}$.
6. 25% of a number is 12. Thus the number itself is $4 \cdot 12$, or 48. 37.5% percent of 48 is then $48 \times .375$ or $\boxed{18}$.
7. There are $7 \cdot 3 = 21$ unit squares in the first rectangle, and $2 \cdot 11 = 22$ unit squares in the second, so there are $21 + 22 = \boxed{43}$ 1×1 squares in total.
8. A pencil costs 30 cents, and an eraser costs 111 cents. So, 5 pencils and 3 erasers cost $5 \cdot 30 + 111 \cdot 3 = \boxed{483}$ cents.
9. For the first 3 hours of his 5 hours, he catches 2 fish per hour, so he catches $3 \cdot 2 = 6$ fish. For the latter 2 hours, he catches 1 fish every hour, so he catches $2 \cdot 1 = 2$ fish. In total, he will catch $6 + 2 = \boxed{8}$ fish.
10. Since he can swallow 2 or 3 mice at a time, and must swallow the 17 mice in the least number of gulps, it is optimal to swallow 3 mice at time as much as possible. thus, if he swallows 3 mice a time 5 times, and 2 mice at time one time, he will have swallowed 17 mice. This takes $5 + 1 = \boxed{6}$ gulps.
11. To find out how many minutes are in a week, note that there are 60 minutes in an hour, 24 hours in a day, and 7 days in a week. Therefore, a week has $60 \times 24 \times 7 = 10080$ minutes. Since Arthur can make a friend every 3 minutes, he can make a maximum of $10080 \div 3 = \boxed{3360}$ friends.
12. There are $\boxed{3}$ primes between 40 and 50. They are 41, 43, and 47.
13. Jay Pee has 2 choices of texture, 4 choices of dye, and 2 choices of length. In total, therefore, he has $2 \cdot 4 \cdot 2 = \boxed{16}$ choices of hairstyle, since for each different outfit he can choose exactly one of each texture, dye, and length.
14. $54\text{slaps} \cdot \frac{4\text{sleps}}{3\text{slaps}} \cdot \frac{5\text{slips}}{8\text{sleps}} \cdot \frac{13\text{slops}}{9\text{slips}} = \boxed{65}$ slops.
15. It is impossible for two different circles to intersect any more than $\boxed{2}$ times.
16. $|8x + 1| = 17$ means that either $8x + 1 = 17$ or $8x + 1 = -17$. Finding the solutions to both of these, $x = 2$ or $\frac{-9}{4}$. These are both perfectly valid, so there are $\boxed{2}$ solutions.
17. The number of sweets Amy has must be a perfect square, since we can arrange the sweets in a square. It must also be divisible by 4 and 5, since we can arrange the sweets in that many rows. Since the number is a perfect square, $5^2 = 25$ must also divide it, and so $4 \cdot 25 = \boxed{100}$ is the least such number.
18. Since the letters that the crowd chant repeat every three letters, every third position will be "A". 2013 is divisible by 3, so the 2013th position will be \boxed{A} .
19. We know that if n is an even number, then $a_n = 1$ and $a_{n+1} = -1$. So for any even n , $a_n + a_{n+1} = 1 + (-1) = 0$. We can thus simplify $a_0 + a_1 + \cdots + a_{2013} = (a_0 + a_1) + (a_2 + a_3) + \cdots + (a_{2012} + a_{2013}) = 0 + 0 + \cdots + 0 = \boxed{0}$.

20. Since $x_n = 1 + \frac{1}{x_{n-1}}$, $x_1 = 1 + \frac{1}{x_0} = \frac{4}{3}$. By the same reasoning, $x_2 = 1 + \frac{1}{\frac{4}{3}} = \frac{7}{4}$, $x_3 = \frac{11}{7}$, and $x_4 = \boxed{\frac{18}{11}}$.
21. The area of the square is $d^2 = 2^2 = 4$. Next, we see that the diameter of the inscribed circle is equal to the side length of the square. So, the radius is equal to $r = \frac{d}{2} = \frac{2}{2} = 1$, and so the area of the inscribed circle is $r^2\pi = 1^2\pi = \pi$. Hence, the area in the square but not in the circle is $\boxed{4 - \pi}$.
22. The possible sums from the two dice rolls are 2, 3, ..., 12. Out of these numbers, only 4, 6, 8, 9, 10, 12 are composite. There are three ways of getting a 4 as the sum—namely, if we roll 1 and 3, 2 and 2, or 3 and 1. Similarly we can also see that there are 5 ways of getting a 6, 5 ways of getting an 8, 4 ways of getting a 9, 3 ways of getting a 10, and 1 way of getting a 12. Lastly, since each die has 6 sides, there are $6 \cdot 6 = 36$ total possible outcomes. Therefore, the answer is $\frac{3 + 5 + 5 + 4 + 3 + 1}{36} = \frac{21}{36} = \boxed{\frac{7}{12}}$.
23. Note that Kelvin claims Alex ate the cake. If he is telling the truth, then both Alex and AJ must be lying. Since there is exactly one liar, Kelvin must be telling a lie. Therefore AJ is telling the truth, and $\boxed{\text{Kelvin}}$ ate the cake.
24. The sum of the numbers from -2013 to 2013 is 0, since for every integer k we are also adding the opposite number $-k$. Next, the sum of the numbers 2014, 2015, and 2016 is 6045. Thus, the sum of the numbers from -2013 to 2016 is 6045, meaning that $n = \boxed{2016}$.
25. Note that if n is in the set, so is $-n$, and so the sum of the numbers in the set is equal to 0. Therefore, the average is also $\boxed{0}$.
26. Josephine eats a quarter of the 48 jelly beans. Thus, she eats 12 of them, leaving 36. She now spills $\frac{2}{3}$ of 36, or 24 of them. This leaves $36 - 24 = \boxed{12}$ for Jared.
27. Since ABC and DEF are similar, $\frac{AB}{BC} = \frac{DE}{EF}$. Therefore, $\frac{3}{4} = \frac{5}{EF} \implies EF = \boxed{\frac{20}{3}}$.
28. The dimensions of the sheet of paper can be converted to 36 inches by 60 inches. So, if both the sheet of paper and the cards are positioned vertically, we can completely cover the paper by placing 6 cards across and 15 cards down. Therefore, our desired answer is $6 \times 15 = \boxed{90}$.
29. Since 374 is even, it is divisible by 2. And since $\frac{374}{2} = 187 = 11 \cdot 17$, we see that 374 can be written as $2 \cdot 11 \cdot 17$. (Note that $2 + 11 + 17 = 30$.) The difference between the smallest and the largest is $17 - 2 = \boxed{15}$.
30. We count off numbers from 99. 99 is only divisible by two primes, 3 and 11. 98 is only divisible by 2 primes, 2 and 7. 97 is prime. 96 is only divisible by two primes, 2 and 3. 95 is only divisible by two primes, 5 and 19. 94 is only divisible by two primes, 2 and 47. 93 is only divisible by two primes, 3 and 31. 92 is only divisible by 2 primes, 2 and 23. 91 is only divisible by 2 primes, 7 and 13. 90 is divisible by three primes, 2, 3, and 5. Thus $\boxed{90}$ is our answer.
31. There is an equal number of quarters, dimes and nickels totaling \$4.80. Let the number of quarters be n . Then $0.25n + 0.1n + 0.05n = 0.4n = 4.80 \implies n = 12$. Since there are $3n$ coins, there are $\boxed{36}$ coins in total in the jar.
32. Let $x = 0.201320132013\dots$. Then, we also get that $10000x = 2013.20132013\dots = 2013 + 0.2013\dots = 2013 + x$. Solving this for x yields $x = \frac{2013}{9999} = \boxed{\frac{61}{303}}$.

33. We need each camera to record an equal number of minutes and we can record a maximum of 60 minutes per camera. $6\frac{1}{2}$ hours is equal to $6.5 \cdot 60 = 390$ minutes. $390 = 2 \cdot 3 \cdot 5 \cdot 13$, so the greatest number less than 60 dividing 390 evenly is $3 \cdot 13 = 39$, and so with each camera we can record 39 minutes. Therefore, we need at least $2 \cdot 5 = \boxed{10}$ cameras.

34. Let S be the sum of the remaining numbers. Since their average is 2013, we get that $\frac{S}{2012} = 2013 \implies S = 2012 \cdot 2013$. So, since the sum of the numbers in the original collection is just $2013 + S$, we get that the original average is

$$\frac{2013 + S}{2013} = \frac{1 \cdot 2013 + 2012 \cdot 2013}{2013} = \frac{2013 \cdot (1 + 2012)}{2013} = \boxed{2013}.$$

35. Dividing 1000 by $5 \cdot 11$ gives 18 and a remainder. $18 \cdot 55 = 990 < 1000$, but 990 is divisible by 3. Therefore, the answer we seek is $990 - 55 = \boxed{935}$, as 935 is not divisible by 3.

36. After drawing the diagram, we see that $CE = CD - DE = 10 - 2 = 8$. Next, by the Power of a Point Theorem, $AE \cdot EB = CE \cdot DE = 8 \cdot 2 = 16$. Since $AE = EB$, $AE^2 = 16 \implies AE = EB = \sqrt{16} = 4$. Therefore, $AB = AE + EB = 4 + 4 = \boxed{8}$.

37. In the prime factorization of a perfect square, each prime factor must have an even exponent. Since $8! = 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 = 2^7 \cdot 3^2 \cdot 5 \cdot 7$. The minimum number that needs to be multiplied to this to result in a factorization with all even exponents is $2 \cdot 5 \cdot 7 = \boxed{70}$.

38. Assume without loss of generality that circle A has a smaller or equal radius than circle B . Call the center of A point D , and the center of B point E . Then $DE = a + b$, since the two circles are externally tangent. Draw the perpendicular to line segment NE from D . Call the foot of the perpendicular F . Then, since $MNFD$ is a rectangle, we get that $EF = EN - NF = EN - DM = b - a$. Lastly, by the Pythagorean Theorem on right triangle DEF ,

$$DF = \sqrt{DE^2 - EF^2} = \sqrt{(a + b)^2 - (b - a)^2} = 2\sqrt{ab} = 2\sqrt{17}.$$

Thus $4ab = 17$, and so $ab = \boxed{\frac{17}{4}}$.

39. We note that the numbers of katz and frogs increase exponentially per year. So, Kelvin will have $80 \cdot 3^n$ katz after n years, and Alex will have $405 \cdot 2^n$ frogs after n years. Therefore, to find the number of years that needs to pass for Kelvin and Alex to have the same number of katz as frogs is the solution to the equation $80 \cdot 3^n = 405 \cdot 2^n$. We can simplify this to the equation $\frac{16}{81} = \frac{2^4}{3^4} = \left(\frac{2}{3}\right)^n$, which yields $n = \boxed{4}$ as the solution.

40. On each edge of the $10 \times 10 \times 10$ cube, there are 10 cubes. The first and last cubes are corner cubes, and have 3 faces painted. The middle 8 have two faces painted. There are 8 such cubes per edge and 12 edges, thus 96 cubes in total. The only other cubes are the cubes part of the 8×8 center square on each face, which only have one face painted. Thus, the answer is $\boxed{96}$.

41. There are 9 digits to be written from 1 to 9. There are $2 \cdot 90 = 180$ digits from 10 to 99. There are $3 \cdot 900 = 2700$ digits written from 100 to 999, which is too large. Therefore the number of pages in the book is some three-digit number. Since there are 2013 digits in the book, we have $2013 = 9 + 180 + 3n$ where n denotes the number of 3-digit numbers in the book. Solving yields $n = 608$ 3-digit pages. Therefore, there are $99 + 608 = \boxed{707}$ pages in the book.

42. If 1 gosling is born out of every 102 duck eggs, then $10404 \div 102 = 102$ goslings are born out of 10404 duck eggs. One-third of these is an ugly duckling, so there are $102 \div 3 = \boxed{34}$ ugly ducklings.

43. When Johnny bought the y mugs, he spent $\$10y$ and was $\$15$ short, meaning that $10y = x + 15$. When he put back half the mug, thus spending a total of $\$5y$, he had $\$25$ left over, so $5y = x - 25$. Subtracting the second equation from the first gives $5y = 40 \implies y = 8$. Plugging this value for y into the first equation results in $80 = x + 15 \implies x = 65$. The sum of x and y is therefore $8 + 65 = \boxed{73}$.
44. We remember that in a regular polygon with n , side, each interior angle has a measure of $\frac{180(n-2)}{n} = \frac{180n-360}{n} = 180 - \frac{360}{n}$. Since the exterior angle is the angle supplementary to this interior angle, we get that the exterior angles all have a measure of $180 - (180 - \frac{360}{n}) = \frac{360}{n}$. Specifically, in the problem we are given that they are each 5 degrees, and so $\frac{360}{n} = 5$. Thus, $n = \boxed{72}$.
45. Let the sum of the numbers on the 8 corners be S . When we take the sum of the numbers on the 6 faces, we see that we counted the number on each vertex 3 times, since a vertex lies on exactly 3 faces. Therefore, $S = \frac{2013}{3} = \boxed{671}$.
46. Arthur can sow a field in 3 hours, so his rate of sowing is $\frac{1}{3}$ per hour. James can sow a field in 5 hours, so his rate of sowing is $\frac{1}{5}$ per hour. Let the number of hours in which Kevin can sow a field be n hours. Then his rate is $\frac{1}{n}$, and their combined rate is $\frac{1}{3} + \frac{1}{5} + \frac{1}{n}$. Since we are given that together they can finish in $\frac{54}{60} = \frac{9}{10}$ hours, we get that the rate combined is also equal to $\frac{1}{3} + \frac{1}{5} + \frac{1}{n} = \frac{1}{\frac{9}{10}} = \frac{10}{9}$. Solving yields $n = \boxed{\frac{45}{26}}$.
47. The formula for T_n can be derived as follows: $T_n + T_n = (1+n) + (2+(n-1)) + \dots + (n+1) = n(n+1) \implies T_n = \frac{n(n+1)}{2}$. (This is the famous formula for the sum of the first n positive integers.) Therefore, $\sqrt{T_{2012} + T_{2013}} = \sqrt{\frac{2012 \cdot 2013}{2} + \frac{2013 \cdot 2014}{2}} = \sqrt{\frac{2013}{2} \cdot 2013 \cdot 2} = \boxed{2013}$.
48. We consider two cases.
- Case 1 is that Arthur finished second. Since James finished before Dennis, the possibilities are James-Arthur-Dennis-Wang, James-Arthur-Wang-Dennis, and Wang-Arthur-James-Dennis, giving a total of 3.
 - Case 2 is that Arthur finished third. The possibilities are then James-Dennis-Arthur-Wang, Wang-James-Arthur-Dennis, and James-Wang-Arthur-Dennis, giving a total of 3.
- Therefore, there are $3 + 3 = \boxed{6}$ ways the race could have ended.

49. If we want to find the probability that event A happens given that event B happens, we have to divide the probability of both events happening by the probability of event B happening. (This is how we can calculate such "conditional" probabilities.) So, in this case, event A is picking box A , and event B is removing a green ball.

Since we choose either box with equal probability, we see that the chance of both choosing box A and choosing a green ball from there is equal to $\frac{1}{2} \cdot \frac{7}{12}$. However, there are actually two ways to choose a green ball—by either picking box A or by picking box B . The first case has the same probability $\frac{1}{2} \cdot \frac{7}{12}$, but the second has probability $\frac{1}{2} \cdot \frac{2}{5}$. Therefore, our answer is equal to

$$\frac{\text{Chance of A and B}}{\text{Chance of B}} = \frac{\frac{1}{2} \cdot \frac{7}{12}}{\frac{1}{2} \cdot \frac{7}{12} + \frac{1}{2} \cdot \frac{2}{5}} = \boxed{\frac{35}{59}}$$

50. Suppose there are a red marbles, b blue marbles, c green marbles, and d orange marbles. Then we have the system of equations

- $b + c + d = 15$
- $a + c + d = 20$
- $a + b + d = 25$
- $a + b + c = 27$.

Adding these gives $3(a + b + c + d) = 87$, and so $a + b + c + d = 29$. Subtracting this equation by the first equation yields $a = \boxed{14}$.