

1. By counting, there are $\boxed{7}$ words in this question.
2. $*(3, 1, 2) = \frac{3^2}{1} + \frac{1^2}{2} + \frac{2^2}{3} = 9 + \frac{1}{2} + \frac{4}{3} = \boxed{\frac{65}{6}}$.
3. A square has 4 sides, so $S = 4$. A pentagon has 5 vertices, so $P = 5$. Hence, $S + P = \boxed{9}$.
4. $1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1}}} = 1 + \frac{1}{1 + \frac{1}{2}} = 1 + \frac{1}{\frac{3}{2}} = 1 + \frac{2}{3} = \boxed{\frac{5}{3}}$.
5. 25% of a number is 12. Thus the number itself is $4 \cdot 12$, or 48. 37.5% percent of 48 is then $48 \times .375$ or $\boxed{18}$.
6. There are $\boxed{3}$ primes between 40 and 50. They are 41, 43, and 47.
7. Let the cost of each box be b and each McNugget be m . Then with the given information, the following two equations may be written: $b + 10m = 4.69$ and $b + 20m = 4.99$. Subtracting the first equation from the second yields $10m = 0.30$ so $m = 0.03$. Plugging this value back into the first equation gives $b + 10(0.03) = 4.69 \implies b = 4.39$. Therefore, a box at McDonald's costs $\boxed{\$4.39}$.
8. Let John's original number be n . Then $\frac{n+10}{5} - 3 = 0 \implies \frac{n+10}{5} = 3 \implies n+10 = 15 \implies n = \boxed{5}$.
9. It is impossible for two different circles to intersect any more than $\boxed{2}$ times.
10. $|8x + 1| = 17$ means that either $8x + 1 = 17$ or $8x + 1 = -17$. Finding the solutions to both of these, $x = 2$ or $\frac{-9}{4}$. These are both perfectly valid, so there are $\boxed{2}$ solutions.
11. The Triangle Inequality states that given a triangle, no side length may equal or exceed the sum of any other two side lengths. In this particular triangle, that means the third length cannot be $5 + 7 = 12$ or greater, so the largest possible value for the length is 11. Similarly, if the side length is 2 or smaller, the sum of this length and the length of 5 will exceed 7. Thus, the side length cannot be any smaller than 3. The difference between the largest and smallest possible values is then $11 - 3 = \boxed{8}$.
12. If n is the answer to this question, then $3n + 12 = n \implies 2n = -12 \implies n = \boxed{-6}$.
13. Note that $x \cdot y + y \cdot x = 2x \cdot y$. Thus, $\frac{(x \cdot y + y \cdot x)^2}{x^2 \cdot y^2} = \frac{4x^2 \cdot y^2}{x^2 \cdot y^2} = \boxed{4}$.
14. This number may be expressed as $3a^2 = 2b^3$ for positive integers a, b . Since 2 is not divisible by 3, 3 must be a factor of b^3 . The smallest b for which this is satisfied is $b = 3$, so $3a^2 = 2(3)^3 = 54 \implies a^2 = 18$, which does not yield an integer value for a . The next smallest value of b that could work is $b = 6$, which gives $3a^2 = 2(6)^3 \implies a^2 = 144$, which does work since a can be 12. Therefore, the smallest positive number that is both thrice a square and twice a cube is $2(6)^3 = 3(12)^2 = \boxed{432}$.
15. The possible sums from the two dice rolls are 2, 3, ..., 12. Out of these numbers, only 4, 6, 8, 9, 10, 12 are composite. There are three ways of getting a 4—namely, if we roll 1 and 3, 2 and 2, or 3 and 1. Similarly we can also see that there are 5 ways of getting a 6, 5 ways of getting an 8, 4 ways of getting a 9, 3 ways of getting a 10, and 1 way of getting a 12.
Lastly, since each die has 6 sides, there are $6 \cdot 6 = 36$ total possible outcomes. Therefore, the answer is $\frac{3 + 5 + 5 + 4 + 3 + 1}{36} = \frac{21}{36} = \boxed{\frac{7}{12}}$.

16. We observe that for any x ,

$$(x-1)^3 = (x^2 - 2x + 1)(x-1) = x^3 - 3x^2 + 3x - 1.$$

(This is an example of the **Binomial Theorem**.) So, plugging in $x = 12$ yields

$$12^3 - 3 \cdot 12^2 + 3 \cdot 12 - 1 = (12-1)^3 = 11^3 = \boxed{1331}.$$

17. Note that Kelvin claims Alex ate the cake. If he is telling the truth, then both Alex and AJ must be lying. Since there is exactly one liar, Kelvin must be telling a lie. Therefore AJ is telling the truth, and Kelvin ate the cake.

18. Josephine eats a quarter of the 48 jelly beans. Thus, she eats 12 of them, leaving 36. She now spills $\frac{2}{3}$ of 36, or 24 of them. This leaves $36 - 24 = \boxed{12}$ for Jared.

19. Let the perimeter of the square and equilateral triangle be $12x$. Thus, each side of the square has length $3x$, while each side of the equilateral triangle has length $4x$. Since the area of a square with side length s is s^2 , the area of the square is thus $9x^2$. Also, since the area of an equilateral triangle with side length s is equal to $\frac{s^2\sqrt{3}}{4}$, the area of the equilateral triangle is $\frac{(4x)^2\sqrt{3}}{4} = 4x^2\sqrt{3}$. The ratio of the area of the triangle to the square is thus $\frac{4x^2\sqrt{3}}{9x^2} = \boxed{\frac{4\sqrt{3}}{9}}$.

20. If the radius is 3 feet, the circumference of each wheel is $2\pi r = 6\pi$. Therefore, the wheels make a total of $150\pi \div 6\pi = \boxed{25}$ rotations.

21. We wish to find how long it takes for the room to cool by $85 - 30 = 55$ degrees. Since Air Conditioner A cools the room by 2 degrees per minute, it cools the room by 4 every two minutes. Thus, every two minutes, the two air conditioners cool the room by a total of $3 + 4 = 7$ minutes. It will then take $\frac{55}{7}$ groups of two minutes, or $2 \cdot \frac{55}{7} = \boxed{\frac{110}{7}}$ minutes to cool the room as desired.

22. The dimensions of the sheet of paper can be converted to 36 inches by 60 inches. So, if both the sheet of paper and the cards are positioned vertically, we can completely cover the paper by placing 6 cards across and 15 cards down. Therefore, our desired answer is $6 \times 15 = \boxed{90}$.

23. If the first car travels at 60 mph, then it required 60 minutes to travel 60 miles. Since the second car moved 40 miles in that time, the second car is traveling at 40 mph. This means every hour, the first car becomes $60 - 40 = 20$ miles closer to the second car, so it will take a total of $\frac{60}{20} = \boxed{3}$ hours for the first car to fully catch up.

24. Let the common ratio of the geometric sequence be x . Thus, since the first term is 81 and the fourth term is 24, we have $81x^3 = 24$ which can be simplified to $x^3 = \frac{8}{27}$. Thus, $x = \frac{2}{3}$, and so the sixth term is $81 \cdot \left(\frac{2}{3}\right)^5 = \boxed{\frac{32}{3}}$.

25. Note that the maximum sum of any two numbers from 1 to 25 is just $25 + 24 = 49$. Thus, the sum of any two adjacent numbers must be 1, 4, 9, 16, 25, 36, or 49. So, the pair of adjacent numbers including 18 must sum to either 25, 36, or 49. However, they can't sum to 36 as $36 = 18 + 18$ and a number can't be repeated. Also, they can't sum to 49 as $49 = 18 + 31$ and 31 doesn't fall between 1 and 25. However, $25 = 18 + 7$, and so the only number that can be next to 18 is 7.

26. Consider $\triangle ABC$, with the vertices being Cities A, B, and C, respectively. We are given that $AC = 100$, $AB + BC = 260$, and $BA = BC$. So, we have that $AB = BC = 130$, and so $\triangle ABC$ is an isosceles triangle with side lengths 100, 130, 130. We see that the problem asks us to find the length of the altitude from B to AC .

Since the triangle is isosceles, the altitude bisects side AC into two segments of length 50. If the altitude hits AC at D , we have that $AD = DC = 50$. But then, $\triangle ADB$ is a right triangle with right angle at D . So by the Pythagorean Theorem we get that

$$BD^2 + AD^2 = BA^2,$$

which implies that

$$BD = \sqrt{BA^2 - AD^2} = \sqrt{130^2 - 50^2} = \sqrt{14400} = \boxed{120}.$$

27. Because 9 people do neither, there are $57 - 9 = 48$ people who swim or play tennis. However, there are a total of $38 + 39 = 77$ people that do both. Thus, if x denotes the number of people who do both, by the Principle of Inclusion Exclusion we have that $77 - x = 48$. Hence, our answer is $77 - 48 = \boxed{29}$.

28. Note that we can split the problem into two cases.

- Case 1: He plays with his eyes open. The probability that Kevin plays with his eyes open is $\frac{2}{3}$. Now because Kevin wins all the games he plays with his eyes open the probability of winning in this case is $\frac{2}{3} * 1 = \frac{2}{3}$.
- Case 2: He plays with his eyes closed. The probability that Kevin plays with his eyes close is $\frac{1}{3}$. The probability that he wins with his eyes closed is $\frac{5}{8}$ so the total probability or winning with his eyes closed is just $\frac{1}{3} * \frac{5}{8} = \frac{5}{24}$.

Therefore, the total probability of winning is just the sum which is $\frac{2}{3} + \frac{5}{24} = \boxed{\frac{7}{8}}$.

29. Let x be the highest score that Josh can get on his final exam. He can maximize x by getting exactly 87% as his average. So, since the final exam is worth 20% of the final average, we have the equation $95 \cdot 0.8 + x \cdot 0.2 = 87$. Solving this equation we get that $x = \boxed{55\%}$.

30. By looking at an odd $n \times n$ square, we see that there are n tiles in each diagonal, giving us a total of $2n - 1$ tiles in both diagonals. Since there are 2013 tiles in the diagonals of this floor, we get that $n = 1007$. Hence, the total number of tiles is $n^2 = \boxed{1014049}$.

31. On each edge of the $10 \times 10 \times 10$ cube, there are 10 cubes. The first and last cubes are corner cubes, and have 3 faces painted. The middle 8 have two faces painted. There are 8 such cubes per edge and 12 edges, thus 96 cubes in total. The only other cubes are the cubes part of the 8×8 center square on each face, which only have one face painted. Thus, the answer is $\boxed{96}$.

32. In the prime factorization of a perfect square, each prime factor must have an even exponent. Since $8! = 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 = 2^7 \cdot 3^2 \cdot 5 \cdot 7$. The minimum number that needs to be multiplied to this to result in a factorization with all even exponents is $2 \cdot 5 \cdot 7 = \boxed{70}$.

33. Let $EC = x$. By definition we have that $DE = DC - EC = 5 - x$. Next, since $\triangle ADE$ is a right triangle, by the Pythagorean Theorem we have that

$$(5-x)^2 + (2x)^2 = 25 \implies 5x^2 - 10x + 25 = 25 \implies x(x-2) = 0 \implies x = 2 \implies DE = 5-2 = \boxed{3}.$$

34. There are 9 digits to be written from 1 to 9. There are $2 \cdot 90 = 180$ digits from 10 to 99. There are $3 \cdot 900 = 2700$ digits written from 100 to 999, which is too large. Therefore the number of pages in the book is some three-digit number. Since there are 2013 digits in the book, we have $2013 = 9 + 180 + 3n$ where n denotes the number of 3-digit numbers in the book. Solving yields $n = 608$ 3-digit pages. Therefore, there are $99 + 608 = \boxed{707}$ pages in the book.

35. Note $a_0 = 2, a_1 = 3, a_2 = 3a_1 - 2a_0 = 5 = 2^2 + 1, a_3 = 3a_2 - 2a_1 = 9 = 2^3 + 1$. We see that one possibility is $a_n = 2^n + 1$. We see that we are right, since we have

$$3(2^{n-1} + 1) - 2(2^{n-2} + 1) = 3 \cdot 2^{n-1} + 3 - 2^{n-1} - 2 = 2 \cdot 2^{n-1} + 1 = 2^n + 1.$$

And so, $a_{10} = 2^{10} + 1 = \boxed{1025}$.

36. We can simply count by cases. There are clearly $9 \cdot 9 = 81$ 1×1 squares. Looking at 2×2 squares, we see that in any row of size 9 one could fit $9 - (2 - 1) = 8$ 2×2 squares across, and analogously one could fit 8 squares down in any column. Thus, there are $8 \cdot 8 = 64$ 2×2 squares. Continuing in this pattern we get that there are $7 \cdot 7 = 49$ 3×3 squares, and so on. Therefore, we have that the total number of squares is equal to $9^2 + 8^2 + \dots + 1^2$. And using the formula $1^2 + 2^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}$, we get the answer to be $\frac{9 \cdot 10 \cdot 19}{6} = \boxed{285}$.

37. Note that $1 + \frac{1}{n} = \frac{n+1}{n}$. Thus, we get that

$$\left(1 + \frac{1}{1}\right) \left(1 + \frac{1}{2}\right) \dots \left(1 + \frac{1}{2013}\right) = \frac{2}{1} \cdot \frac{3}{2} \cdot \frac{4}{3} \dots \frac{2014}{2013} = \frac{2014}{1} = \boxed{2014},$$

since all of the numerators and denominators except for the first and last cancel.

38. The diagonal of this rectangle will be the diameter of the circle. Since the side lengths are 6 and 8, by the Pythagorean Theorem, the diameter will be $\sqrt{6^2 + 8^2} = 10$. The area of the circle is then $\pi r^2 = \pi \left(\frac{10}{2}\right)^2 = \boxed{25\pi}$.

39. We consider two cases.

- Case 1 is that Arthur finished second. Since James finished before Dennis, the possibilities are James-Arthur-Dennis-Wang, James-Arthur-Wang-Dennis, and Wang-Arthur-James-Dennis, giving a total of 3.
- Case 2 is that Arthur finished third. The possibilities are then James-Dennis-Arthur-Wang, Wang-James-Arthur-Dennis, and James-Wang-Arthur-Dennis, giving a total of 3.

Therefore, there are $3 + 3 = \boxed{6}$ ways the race could have ended.

40. Clearly only the number 1 has exactly one factor. Also, if $n > 1$ is an integer, then we know that 1 and n are already factors. Moreover, if p is a prime factor of n , then $\frac{n}{p}$ is also a factor of n since $n \div \frac{n}{p} = p$. Therefore, if n has exactly three divisors, we must have $p = \frac{n}{p}$, meaning that $n = p^2$. This shows that the only numbers with exactly three divisors are squares or primes. By counting, we see that there are 14 primes whose squares are less than 2013 - the largest of which is $43^2 = 1849$. So our answer is $14 + 1 = \boxed{15}$.

41. The sum of the columns and rows takes into account every entry twice, so no matter what arrangement the numbers are in, the sum of all the columns and all the rows is $2(1+2+\dots+9) = 90$. Thus, the sum of the diagonals is $124 - 90 = \boxed{34}$.

42. First, note that both sides have to be divisible by 9. Taking the sum of the digits, we get that it is $45 + A$. By the divisibility rule for 9, we get that $45 + A$ is divisible by 9, so $A = 0$ or $A = 9$. Next, also note that both sides have to be divisible by 11. Using the divisibility rule for 11 we have that $1 + A + 6 + 4 + 6 - 3 - 7 - 7 - 3 - 8 = A - 11$ must be divisible by 11. So $A = 0, 11$ but since 11 can't be a digit, $A = \boxed{0}$.

43. In order to return to its original position, the bug must move right 3 times and left 3 times in some order, for a total of 6 moves. There are $\binom{6}{3} = 20$ ways to move this way. Also, there are $2^6 = 64$ total possible moves since for each move the bug can move left or right. Thus, our answer is $\frac{20}{64} = \boxed{\frac{5}{16}}$.
44. Note that since $AB = 3$ and since 3 is the radii of Circle O , we have that triangle OAB is equilateral and that $\angle AOB = 60^\circ$. So, the area of half of the intersection is just $\frac{60^\circ}{360^\circ} = \frac{1}{6}$ of the area of the circle minus the area of triangle OAB . Remembering that the area of an equilateral triangle with side length s is equal to $\frac{s^2\sqrt{3}}{4}$, we can get that the area of half of the intersection is just $\frac{1}{6}(3^2\pi) - \frac{3^2\sqrt{3}}{4} = \frac{3}{2}\pi - \frac{9\sqrt{3}}{4}$. We can now double this number to get the final answer of $\boxed{3\pi - \frac{9\sqrt{3}}{2}}$.
45. The formula for T_n can be derived as follows: $T_n + T_n = (1+n) + (2+(n-1)) + \dots + (n+1) = n(n+1) \implies T_n = \frac{n(n+1)}{2}$. (This is the famous formula for the sum of the first n positive integers.)
Therefore, $\sqrt{T_{2012} + T_{2013}} = \sqrt{\frac{2012 \cdot 2013}{2} + \frac{2013 \cdot 2014}{2}} = \sqrt{\frac{2013}{2} \cdot 2013 \cdot 2} = \boxed{2013}$.
46. Suppose there are a red marbles, b blue marbles, c green marbles, and d orange marbles. Then we have the system of equations
- $b + c + d = 15$
 - $a + c + d = 20$
 - $a + b + d = 25$
 - $a + b + c = 27$.
- Adding these gives $3(a + b + c + d) = 87$, and so $a + b + c + d = 29$. Subtracting this equation by the first equation yields $a = \boxed{14}$.
47. We see that the beginning of the problem is completely irrelevant to the problem, since the only relevant information is the number of coins flipped. Note that for any number of coins, the probability of getting more heads than tails is equal to the probability of getting more tails than heads, since the two cases are completely symmetric. Therefore, both events have a probability of $\frac{1}{2}$ since they are opposite events, meaning that the chance of Kelvin the Frog winning is $\boxed{\frac{1}{2}}$.
48. The method to solving this problem is known as circles and bars. Imagine that there are 10 cookies as circles in a line and that there are 2 bars. Note that wherever you place the two bars, each arrangement represents a unique way to split 10 cookies among three flavors. For example, any circle that is to the left of the leftmost bar is say chocolate chip. Any circle that is to the right of the rightmost bar is say peanut butter. Any circle in the middle of the two bars is sugar. Thus the number of ways to split 10 cookies among 3 flavors is just the same as the number of ways to arranged 10 circles and 2 bars in a line, which is just $\binom{12}{2} = \boxed{66}$.
49. Note that the Angle Bisector Theorem states in this case that $\frac{AB}{AD} = \frac{CB}{CD}$. If $x = AD$, then we get that $DC = 7 - x$ and so $\frac{6}{x} = \frac{8}{7-x}$. Solving this equation yields $AD = 3$ and that $CD = 4$. Now, Stewart's Theorem states that in this case $4 \cdot 3 \cdot 7 + BD^2 \cdot 7 = 8^2 \cdot 3 + 6^2 \cdot 4$. This implies that $7BD^2 = 252 \implies BD = 6$. Now again by the Angle Bisector Theorem we have

that $\frac{BD}{BE} = \frac{CD}{CE}$. Letting $y = CE$ we get $BE = 8 - y$ and so $\frac{6}{8 - y} = \frac{4}{y}$. We can thus solve for CE and get that $CE = \boxed{\frac{16}{5}}$.

50. Note that we can rewrite the set in terms of mod 3. (Note: a number "mod" 3 is the remainder when that number is divided by 3.) Thus the set $\{1, 2, 3, \dots, 2013\}$ becomes $\{1, 2, 0, \dots, 1, 2, 0\}$. Note that any combination of a 1 and a 2 will create a number 0 mod 3. Therefore, if a number 1 mod 3 is in the set, then a number which is 2 mod 3 cannot be in the set. Note that one cannot also have 2 numbers which are both 0 mod 3 in the set. Therefore, to maximize the number elements such that the sum of any two chosen elements is not divisible by 3 is just either choosing all the elements to be 1 mod 3 or all elements that are 2 mod 3. Either way you get that since the set cycles by sets of 3 there are 671 such numbers in each case. However, note that one can add an additional element that is 0 mod 3, but no more, giving us a grand total of $\boxed{672}$ elements.