1. Recall that we should calculate from left to right. Hence,

$$5 \div 4 \times 3 \div 2 \times 1 = \left(\frac{5}{4}\right) \times 3 \div 2 \times 1 = \left(\frac{15}{4}\right) \div 2 \times 1 = \left(\frac{15}{8}\right) \times 1 = \boxed{\frac{15}{8}}.$$

2. $*(4,3,2) = \frac{4^2}{3} + \frac{3^2}{2} + \frac{2^2}{4} = \frac{16}{3} + \frac{9}{2} + \frac{4}{4} = \boxed{\frac{65}{6}}$.

3. A square has 4 sides, so $S = 4$. A pentagon has 5 vertices, so $P = 5$. Hence, $S + P = \boxed{9}$.

4. Even numbers greater than 2 are not prime, since 2 divides any even number. So this leaves $111, 113, 115, 117, 119$ as the only possible primes. However, $111 = 3 \cdot 37, 115 = 5 \cdot 23, 117 = 9 \cdot 13$, and $119 = 7 \cdot 17$. Therefore, 113 is the only prime, and so the answer is $\boxed{1}$.

5. It is impossible for two different circles to intersect any more than $\boxed{2}$ times.

6. John has 5 different choices for shorts, 3 for shoes, and 2 for shirts. Since any combination with at least one different article of clothing counts as a different outfit, there are $5 \times 3 \times 2 = \boxed{30}$ different outfits.

7. $|8x + 1| = 17$ means that either $8x + 1 = 17$ or $8x + 1 = -17$. Finding the solutions to both of these, $x = 2$ or $\frac{-9}{4}$. These are both perfectly valid, so there are $\boxed{2}$ solutions.

8. Let the square have side length s. Since the area of a square is equal to s^2 and its perimeter is equal to $4s$, the question tells us that $s^2 = 4s$. Since $s \neq 0$, we get that $s = \boxed{4}$.

9. $54\text{slaps} \cdot \frac{4\text{sleps}}{3\text{slaps}} \cdot \frac{5\text{slips}}{8\text{sleps}} \cdot \frac{13\text{slops}}{9\text{slips}} = \boxed{65}$ slops.

10. To send a bit successfully to computer B, computer A needs to send the correct bit at least twice in the string of three bits. If it sent exactly two correct bits, there are three potential slots where the incorrect bit can come in. (For instance, if the correct bit is 0, the three possible cases would be 001, 010, and 100.) Hence the probability of sending exactly two correct bits is $\frac{2}{3} \cdot \frac{2}{3} \cdot \frac{1}{3} \cdot 3 = \frac{4}{9}$.

    Lastly, the probability of sending three correct bits is equal to $\frac{2}{3} \cdot \frac{2}{3} \cdot \frac{2}{3} = \frac{8}{27}$. Therefore, the final probability is equal to $\frac{4}{9} + \frac{8}{27} = \boxed{\frac{20}{27}}$.

11. Since m is a side length in the new rectangle and $m \neq n$, we see that the original side of length n was folded in half into the side of length p. Hence, because $2n \neq m$, $p = \frac{n}{2}$, and so $\frac{n}{p} = \frac{\frac{n}{1}}{\frac{n}{2}} = \boxed{2}$.

12. Note that $(-1)^{-1} = \frac{1}{(-1)^1} = -1$. So, $i_n = -1$ for all positive integer n, and thus $i_0 + i_1 + \cdots + i_{2013} = (-1) + (-1) + \cdots + (-1) = \boxed{-2014}$.

13. The possible sums from the two dice rolls are $2, 3, \ldots, 12$. Out of these numbers, only $4, 6, 8, 9, 10, 12$ are composite. There are three ways of getting a 4 as the sum– namely, if we roll 1 and 3, 2 and 2, or 3 and 1. Similarly we can also see that there are 5 ways of getting a 6, 5 ways of getting an 8, 4 ways of getting a 9, 3 ways of getting a 10, and 1 way of getting a 12.

    Lastly, since each die has 6 sides, there are $6 \cdot 6 = 36$ total possible outcomes. Therefore, the answer is $\frac{3+5+5+4+3+1}{36} = \frac{21}{36} = \boxed{\frac{7}{12}}$.

14. We observe that for any x,

$$(x-1)^3 = (x^2 - 2x + 1)(x-1) = x^3 - 3x^2 + 3x - 1.$$

(This is an example of the **Binomial Theorem**.) So, plugging in $x = 12$ yields

$$12^3 - 3 \cdot 12^2 + 3 \cdot 12 - 1 = (12-1)^3 = 11^3 = \boxed{1331}.$$

15. We note that $5^2 = 25$, $5^3 = 125$, and $5^4 = 625$, so we can deduce that 5^n for integers $n >= 2$ will always end in 25. Therefore, since t_{2013} is a power of 5 raised to an exponent greater than 2, it must end in $\boxed{25}$.

For a more rigorous solution, consider the following: We will demonstrate a surprising result - that the last two digits of any power of 5 greater than 5 will have be 25. Suppose n is a positive integer with last two digits 25. Then we can write n as $n = 100a + 25$, where a is a nonnegative integer. But then

$$5 \cdot n = 5 \cdot (100a + 25) = 500a + 125 = 500a + 100 + 25 = 100(5a + 1) + 25.$$

So, since $5a + 1$ is a positive integer, the last two digits of $5n$ will also be 25. And since the last two digits of $5^2 = 25$ are 25, we see that the last two digits of

$$5 \cdot 5^2 = 5^3, 5 \cdot 5^3 = 5^4, \ldots$$

will allbe 25. Therefore, any power of 5 greater than 5 will end with 25.

Specifically, since t_{2013} is a power of 5, we see that the answer is $\boxed{25}$.

16. We see that there are 365 days in a non-leap year, and that 365 leaves a remainder of 1 when divided by 7, which is the number of days in a week. Thus, after each non-leap year, we see that October 13th will move ahead by one day. (For instance, in 2014 it will be a Monday.) However, on a leap year, there are 366 days, which leaves a remainder of 2 when divided by 7. So this tells us that in a leap year, October 13th will move ahead by *two* days. Since the year 2016 is a leap year, we see that in 2015 it will be a Tuesday; in 2016, Thursday; in 2017, Friday; in 2018, Saturday; in 2019, Sunday. Thus the answer is $\boxed{2019}$.

17. Note that Kelvin claims Alex ate the cake. If he is telling the truth, then both Alex and AJ must be lying. Since there is exactly one liar, Kelvin must be telling a lie. Therefore AJ is telling the truth, and $\boxed{\text{Kelvin}}$ ate the cake.

18. We observe that

$$!2013! = (-2013)(-2012)\cdots(-1) \cdot 0 \cdot 1 \cdots 2013 = 0.$$

So, its remainder when divided by 2017 is $\boxed{0}$.

19. We see that

$$(a+b)^x = (a^x)(b^x) = (ab)^x.$$

This implies that either $a + b = ab$ or $x = 0$. From the first possibility it follows that

$$\begin{aligned} 0 &= ab - a - b \\ &= ab - a - b + 1 - 1 \\ &= a(b-1) - (b-1) - 1 \\ &= (b-1)(a-1) - 1. \end{aligned}$$

Hence we get that $1 = (a-1)(b-1)$. Sincea, b are integers, we must have that $a-1 = b-1 = \pm 1$. But either case implies that $a = b$, which contradicts the fact that a, b are distinct. Therefore, we cannot have that $a + b = ab$, and so the only possible answer is $x = \boxed{0}$.

20. We see that
$$2025 = 45^2 = (3x^2+1)^2 = 9x^4 + 6x^2 + 1.$$
So, it follows that
$$9x^4 + 6x^2 - 11 = (9x^4 + 6x^2 + 1) - 12 = 2025 - 12 = \boxed{2013}.$$

Alternate Solution: Since $3x^2 + 1 = 45$, we get that $3x^2 = 44$. Substituting this value into the desired expression yields
$$9x^4 + 6x^2 - 11 = (3x^2)^2 + 2(3x^2) - 11 = 44^2 + 2 \cdot 44 - 11 = 1936 + 88 - 11 = \boxed{2013}.$$

21. Suppose a and b are diametrically opposite such that $a < b$. Then there are equal amounts of numbers from a to b on the circle and from b to a. Since there are $n-2$ total numbers excluding a and b, there must be $\frac{n-2}{2}$ numbers from a to b. And since the numbers are placed in order, we have that
$$b - a = \frac{n-2}{2} + 1 = \frac{n}{2}.$$
Specifically, since $a = 17$ and $b = 38$, we see that $b - a = 38 - 17 = 21 = \frac{n}{2}$, and so $n = \boxed{42}$.

22. In the worst case scenario, in the first 17 trials Izzy can take out one sock from each of the 17 pairs, so that she has no matching pairs yet. But then, there is one sock from each pair remaining, and so she can get a pair of matching socks by taking out any of the 17 socks left. Hence, in each of her next two tries she is guaranteed to get a matching pair, and so she can guarantee having two matching pairs within $17 + 2 = \boxed{19}$ socks. More rigorously, this is an example of the **Pigeonhole Principle**. As there are 17 pairs, choosing any 19 of them necessary includes at least 2 matching pairs.

23. Recall that by difference of squares,
$$x^2 - 1 = (x+1)(x-1).$$
We can rearrange this to get
$$x^2 = (x+1)(x-1) + 1.$$
Hence it follows that
$$\left(\sqrt{1 + 17\sqrt{16 \cdot 14 + 1}}\right)^{\frac{1}{4}} = \left(\sqrt{1 + 17\sqrt{15^2}}\right)^{\frac{1}{4}} = \left(\sqrt{1 + 17 \cdot 15}\right)^{\frac{1}{4}} = \left(\sqrt{16^2}\right)^{\frac{1}{4}} = 16^{\frac{1}{4}} = (2^4)^{\frac{1}{4}} = \boxed{2}.$$

24. Consider $\triangle ABC$, with the vertices being Cities A,B, and C, respectively. We are given that $AC = 100, AB + BC = 260$, and $BA = BC$. So, we have that $AB = BC = 130$, and so $\triangle ABC$ is an isosceles triangle with side lengths $100, 130, 130$. We see that the problem asks us to find the length of the altitude from B to AC.

Since the triangle is isosceles, the altitude bisects side AC into two segments of length 50. If the altitude hits AC at D, we have that $AD = DC = 50$. But then, $\triangle ADB$ is a right triangle with right angle at D. So by the Pythagorean Theorem we get that
$$BD^2 + AD^2 = BA^2,$$
which implies that
$$BD = \sqrt{BA^2 - AD^2} = \sqrt{130^2 - 50^2} = \sqrt{14400} = \boxed{120}.$$

25. In the prime factorization of a perfect square, each prime factor must have an even exponent. Since $8! = 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 = 2^7 \cdot 3^2 \cdot 5 \cdot 7$. The minimum number that needs to be multiplied to this to result in a factorization with all even exponents is $2 \cdot 5 \cdot 7 = \boxed{70}$.

26. Because 9 people do neither, there are $57 - 9 = 48$ people who swim or play tennis. However, there are a total of $38 + 39 = 77$ people that do both. Thus, if x denotes the number of people who do both, by the Principle of Inclusion Exclusion we have that $77 - x = 48$. Hence, our answer is $77 - 48 = \boxed{29}$.

27. Note that $\angle ABC = \angle ADB$. Thus, by similar triangles, we have $\dfrac{AC}{BC} = \dfrac{BC}{CD}$, and so $BC^2 = AC \cdot CD$. Thus, $12^2 = 5 \cdot CD$, and $CD = \boxed{\dfrac{144}{5}}$.

28. To cut the log into 6 pieces, Kelvin must cut the log 5 times. Since it thus takes him $\dfrac{20}{5} = 4$ minutes to make one cut, it takes him $4 \cdot 9 = \boxed{36}$ minutes to make 9 cuts.

29. Because $\triangle ABX$ is isosceles, we have $\angle BAX = \angle ABX = 20°$. So, we have $\angle BXA = 180° - 20° - 20° = 140°$. In addition, $\triangle AXC$ is isosceles, and thus $\angle AXB = \angle ACX = 180° - 140° = 40°$. Thus finally, $\angle XAC = 180° - 40° - 40° = \boxed{100°}$.

30. The increase in volume of the water is equivalent to the volume of the rock, and is also equivalent to that of a cylinder with radius and height 2. The volume of the rock is thus $\pi(2^2)(2) = \boxed{8\pi}$ cm^3.

31. There are 9 digits to be written from 1 to 9. There are $2 \cdot 90 = 180$ digits from 10 to 99. There are $3 \cdot 900 = 2700$ digits written from 100 to 999, which is too large. Therefore the number of pages in the book is some three-digit number. Since there are 2013 digits in the book, we have $2013 = 9 + 180 + 3n$ where n denotes the number of 3-digit numbers in the book. Solving yields $n = 608$ 3-digit pages. Therefore, there are $99 + 608 = \boxed{707}$ pages in the book.

32. Note that $1 + \dfrac{1}{n} = \dfrac{n+1}{n}$. Thus, we get that

$$\left(1 + \frac{1}{1}\right)\left(1 + \frac{1}{2}\right)\cdots\left(1 + \frac{1}{2013}\right) = \frac{2}{1} \cdot \frac{3}{2} \cdot \frac{4}{3} \cdots \frac{2014}{2013} = \frac{2014}{1} = \boxed{2014},$$

since all of the numerators and denominators except for the first and last cancel.

33. Suppose the three numbers chosen are a, b, c with $a < b < c$. Then notice that even if a is the largest term less than b and that if c were the least term greater than b, then $a + b = c$. This fails the Triangle Inequality, which says that if a, b, c are the sides of a triangle, then$a + b > c$. So no such numbers work and the probability is $\boxed{0}$.

34. Note $a_0 = 2, a_1 = 3, a_2 = 3a_1 - 2a_0 = 5 = 2^2 + 1, a_3 = 3a_2 - 2a_1 = 9 = 2^3 + 1$. We see that one possibility is $a_n = 2^n + 1$. We see that we are right, since we have

$$3(2^{n-1} + 1) - 2(2^{n-2} + 1) = 3 \cdot 2^{n-1} + 3 - 2^{n-1} - 2 = 2 \cdot 2^{n-1} + 1 = 2^n + 1.$$

And so, $a_{10} = 2^{10} + 1 = \boxed{1025}$.

35. If the side lengths of a rectangular prism are a, b, c, then the diagonal of one face can be expressed as $\sqrt{a^2 + b^2}$. The diagonal of the whole prism can then be expressed as $\sqrt{\left(\sqrt{a^2 + b^2}\right)^2 + c^2} = \sqrt{a^2 + b^2 + c^2} = 13$. Squaring both sides gives $a^2 + b^2 + c^2 = 169$. Since a, b, c are all integers, we wish to find integer values that work in this equation.

Without loss of generality, we can assume that $a \le b \le c$. Thus $a^2 + b^2 + c^2 \le 3c^2$, and so $c^2 \le 56$. Thus we need only check $c \ge 8$.

- If $c = 8$, then $a^2 + b^2 = 105$. Hence b can either be 7 or 8. If $b = 7$, then $a^2 + b^2 \le 2b^2 = 98 < 105$, but if $b = 8$ then $a = \sqrt{41}$, which is not an integer. Hence $c \neq 8$.
- If $c = 9$ then $a^2 + b^2 = 88$. Another routine check shows that $b = 8, 9$ return non-integer values for a.
- If $c = 10$ then $a^2 + b^2 = 69$. Then $b = 6, 7, 8, 9$ again all fail.
- If $c = 11$ then $a^2 + b^2 = 48$. We again verify that there are no solutions.
- If $c = 12$ then $a^2 + b^2 = 25$, which has solution $a = 3, b = 4$.

Thus our volume is $3 \cdot 4 \cdot 12 = \boxed{144}$.

Alternate solution : Suppose $a^2+b^2+c^2 = 13^2$ as above. We first observe that $5^2+12^2 = 13^2$. Since we also recall that $5^2 = 4^2 + 3^2$, we get that $13^2 = 12^2 + 4^2 + 3^2$. Hence the volume is $3 \cdot 4 \cdot 12 = \boxed{144}$.

36. Rearranging the given equation gives $2(a^2 + b^2) = a^2 - b^2 \Rightarrow 2a^2 + 2b^2 = a^2 - b^2 \Rightarrow a^2 = 3b^2$. Therefore squaring both sides yields $a^4 = 9b^4$, and so

$$\frac{a^4 + b^4}{a^4 - b^4} = \frac{9b^4 + b^4}{9b^4 - b^4} = \frac{10b^4}{8b^4} = \boxed{\frac{5}{4}}.$$

37. Suppose there are a red marbles, b blue marbles, c green marbles, and d orange marbles. Then we have the system of equations

- $b + c + d = 15$
- $a + c + d = 20$
- $a + b + d = 25$
- $a + b + c = 27$.

Adding these gives $3(a + b + c + d) = 87$, and so $a + b + c + d = 29$. Subtracting this equation by the first equation yields $a = \boxed{14}$.

38. If we "unfold" the cube to form a flat net, we remember that the shortest distance from one corner to the opposite corner is a straight line. In this case, this line is the hypotenuse of a right triangle with legs 1 and 2, thus the minimum distance is $\boxed{\sqrt{5}}$.

39. The digit 1 will appear in the thousands place exactly 6 times, contributing $6 \cdot 1000 = 6000$ to the sum, the hundreds place 6 times, the tens place 6 times, and the ones place 6 times. The same is true for the other digits. Thus our answer is

$$6 \cdot 1111 + 6 \cdot 2222 + 6 \cdot 3333 + 6 \cdot 4444 = 6 \cdot (1111 + 2222 + 3333 + 4444) = 6(11110) = \boxed{66660}.$$

Alternate Solution: Note that there are 24 possible numbers formed by the given constraint. Now, the average of a set of numbers is the sum of those numbers divided by the number of those numbers, so we can find the sum of all these numbers by multiplying the average value of these by 24. The average of the numbers 1,2,3,4 is 2.5 so a 4-digit number with 2.5 in the ones, tens, hundreds, and thousands place evaluates to 2777.5. $2777.5(24) = \boxed{66660}$, as desired.

40. In order to return to its original position, the bug must move right 3 times and left 3 times in some order, for a total of 6 moves. There are $\binom{6}{3} = 20$ ways to move this way. Also, there are $2^6 = 64$ total possible moves since for each move the bug can move left or right. Thus, our answer is $\frac{20}{64} = \boxed{\frac{5}{16}}$.

41. Clearly $x = 0$ does not satisfy the equation. Thus, we can divide by x to get $x + \frac{1}{x} = 1$. Squaring this equation gives $x^2 + \frac{1}{x^2} + 2 = 1$, and multiplying both sides by x^2 yields $x^4 + x^2 + 1 = 0$. Therefore, $a = \boxed{1}$.

42. Adding the equations gives
$$a^2 + 2ab + b^2 = (a + b)^2 = 256.$$
As a, b are both positive, we have $a + b = 16$. Thus, $a(a + b) = a \cdot 16 = 144$ gives $a = 9$, and $b(a + b) = b \cdot 16 = 112$ gives $b = 7$. Thus $a - b = \boxed{2}$.

43. Note that 10! is a multiple of 100, since it is divisible by $5 \cdot 10 = 500$. Thus it ends in two zeros. However, $9! = 362880$ is not a multiple of 100. Therefore, $n!$ ends in 2 zeros iff $n \geq 10$. As a result it suffices to find the last 2 digits of $9!+8!+\cdots+1!$, which yields $80+20+40+20+20+24+6+2+1 \equiv \boxed{13}$.

44. Divide the array into 2×3 rectangles. Since this is a similar rectangle, the diagonal intersects the opposite endpoints of the 2×3 array. So, in each 2×3 grid, exactly 4 of these squares have any part of the diagonal inside them. As there are 36 such rectangles in a 72×108 array that the diagonal goes through, there are $4 \cdot 36 = \boxed{144}$ such squares. (In general if we consider an $m \times n$ rectangle, try proving that the number of squares intersected is $m + n - \gcd(m, n)$.)

45. Let $\overline{AB} = x$, and extend the lines BC, DE, FA. We see that we form an equilateral triangle with side length $4x$. Since the area of an equilateral triangle with side length s is equal to $\frac{s^2\sqrt{3}}{4}$, the area of the hexagon is then $\frac{(4x)^2\sqrt{3}}{4} - 3(\frac{x^2\sqrt{3}}{4}) = \frac{13x^2\sqrt{3}}{4} = 52\sqrt{3}$. Solving reveals that $x = 4$, and so the perimeter is $x + 2x + x + 2x + x + 2x = 9x = \boxed{36}$.

46. The equation will have two real solutions if and only if the discriminant is positive. The discriminant is $(2a)^2 - 4(b^2)(1) = 4a^2 - 4b^2$, which is positive exactly when $a > b$. But as a and b are distinct, the probability of this is exactly $\boxed{\frac{1}{2}}$ by symmetry.

47. There are 4 cases, in which there are 0, 2, 4, or 6 draws.
   - In the first case there are $\binom{6}{3} = 20$ ways to arrange the 3 wins and 3 losses.
   - In the second case there are $\frac{6!}{2!2!2!} = 90$ ways to arrange the 2 wins, 2 draws, and 2 losses.
   - In the third case there are $\frac{6!}{1!4!1!} = 30$ ways to arrange the 1 win, 4 draws, and 1 loss.
   - Finally, there is 1 way for there to be 6 draws.

   Thus there is a total of $20 + 90 + 30 + 1 = \boxed{141}$ ways for the match to end in a tie.

48. As 0 is in the set, any number that can be expressed in the form $5a + 7b$ for nonnegative numbers a and b is also in the set. By the Chicken McNugget Theorem, the largest number not expressible in this form is $5 \cdot 7 - 5 - 7 = \boxed{23}$.

49. Let there be a_n good subsets of the set $\{1, 2, \ldots, n\}$. If 1 is in the subset, then 2 cannot be in the subset, and there are a_{n-2} ways to choose the remaining numbers in the subset. If 1 is not in the subset, then there are a_{n-1} ways to choose the remaining numbers in the subset. Therefore, $a_n = a_{n-1} + a_{n-2}$. Furthermore, $a_1 = 2$ and $a_2 = 3$. Therefore $a_3 = 5, a_4 = 8, a_5 = 13, a_6 = 21, a_7 = 34, a_8 = 55, a_9 = 89, a_{10} = \boxed{144}$. Note that these numbers follow the Fibonacci sequence.

50. Note that as the ball deflates, the surface area stays the same. Since the basketball originally has a radius of 6 inches, the ball has a surface area of $4\pi(6)^2 = 144\pi$. Let the hemisphere have radius r. Then we have the equation $\pi r^2 + 2\pi r^2 = 3\pi r^2 = 144\pi \implies r = 4\sqrt{3}$. Thus our desired ratio is $\frac{\frac{2\pi(4\sqrt{3})^3}{3}}{\frac{4\pi(6)^3}{3}} = \boxed{\frac{4\sqrt{3}}{9}}$.