1. What is the probability that a randomly chosen word of this sentence has exactly four letters?
2. What's the closest number to 169 that's divisible by 9?
3. Let $a\#b$ be defined as $ab - a - 3$. For example, $4\#5 = 20 - 4 - 3 = 13$. Compute $(2\#0)\#(1\#4)$.
4. 27 students in a school take French. 32 students in a school take Spanish. 5 students take both courses. How many of these students in total take only 1 language course?
5. A palindrome is a word that reads the same backwards as forwards, such as "eye", "race car", and "qwertyytrewq". How many letters are in the smallest palindrome containing the letters b, o, g, t, r, and o, not necessarily in that order and not necessarily adjacent?
6. Alex the Kat has written 61 problems for a math contest, and there are a total of 187 problems submitted. How many more problems does he need to write (and submit) before he has written half of the total problems?
7. How many digits could possibly be the last digit of a perfect square?
8. A hedgehog has 4 friends on Day 1. If the number of friends he has increases by 5 every day, how many friends will he have on Day 6?
9. Let $n! = n \cdot (n-1) \cdot (n-2) \cdot \ldots \cdot 2 \cdot 1$. For example, $5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$. Compute $\frac{(6!)^2}{5! \cdot 7!}$.
10. Find the sum of the greatest common factor and the least common multiple of 12 and 18.
11. What number is exactly halfway between $\frac{1}{6}$ and $\frac{1}{4}$?
12. Lev scores 91, 89, 88, 94, 87, 85 on his first 6 tests. After having his final exam, he (correctly) states that the average of all 7 of his test scores is equal to his final exam score. What was Lev's final exam score?
13. Define $x \star y$ to be $x^y + y^x$. Compute $2 \star (2 \star 2)$.
14. 11 consecutive integers sum to 1331. What is the largest of the 11 integers?
15. Rita the painter rolls a fair 6-sided die that has 3 red sides, 2 yellow sides, and 1 blue side. Rita rolls the die twice and mixes the colors that the die rolled. What is the probability that she has mixed the color purple?
16. The sum of two integers is 8. The sum of the squares of those two integers is 34. What is the product of the two integers?
17. Find all values of x such that $\frac{x^2+1}{x-1} = \frac{x^2-1}{x+1}$.
18. A 6-year stock that goes up 30% in the first year, down 30% in the second, up 30% in the third, down 30% in the fourth, up 30% in the fifth, and down 30% in the sixth is equivalent to a 3-year stock that loses $x\%$ in each of its three years. Compute x.
19. Lev and Alex are racing on a number line. Alex is much faster, so he goes to sleep until Lev reaches 100. Lev runs at 5 integers per minute and Alex runs at 7 integers per minute (in the same direction). How many minutes from the START of the race will it take Alex to catch up to Lev (who is still running after Alex wakes up)?
20. The side length of a cube is increased by 20%. The surface area of the cube then increases by $x\%$ and the volume of the cube increases by $y\%$. Find $5(y - x)$.
21. Kelvin the Frog and Alex the Kat play a game. Kelvin the Frog goes first, and they alternate rolling a standard 6-sided die. If they roll an even number or a number that was previously rolled, they win. What is the probability that Alex wins?
22. For how many positive integer values of x is $4^x - 1$ prime?
23. An isosceles triangle has side lengths $x - 4, 2x - 9$, and $3x - 15$. Find the sum of all possible values of x.
24. When submitting problems, Steven the troll likes to submit silly names rather than his own. On day 1, he gives no name at all. Every day after that, he alternately adds 2 words and 4 words to his name. For example, on day 4 he submits an 8-word name. On day n he submits the 44-word name "Steven the AJ Dennis the DJ Menace the Prince of Tennis the Merchant of Venice the Hygienist the Evil Dentist the Major Premise the AJ Lettuce the Novel's Preface the Core Essence the Young and the Reckless the Many Tenants the Deep, Dark Crevice". Compute n.

25. If a triangle has three altitudes of lengths 6, 6, and 6, what is its perimeter?

26. Alex is training to make MOP. Currently he will score a 0 on the AMC, the AIME, and the USAMO. He can expend 3 units of effort to gain 6 points on the AMC, 7 units of effort to gain 10 points on the AIME, and 10 units of effort to gain 1 point on the USAMO. He will need to get at least 200 points on the AMC and AIME combined **and** get at least 21 points on the USAMO to make MOP. What is the minimum amount of effort he can expend to make MOP?

27. Young Guy likes to make friends with numbers, so he calls a number "friendly" if the sum of its digits is equal to the product of its digits. How many 3 digit friendly numbers are there?

28. Points $D, E, F,$ and G lie outside unit square $ABCD$ such that ADB, BEC, CFD, and DGA are all equilateral triangles. Find the area of square $DEFG$.

29. Lev and Alex are buying New York Giants hats. After purchasing one, they leave a tip for the sales clerk. Lev wants to leave 20% of their bill, while Alex wants to round up to the nearest dollar (so on a bill of $8.36 he would tip 64 cents). What is the largest possible cost, in cents, of a New York Giants hat such that Alex wants to tip more than Lev (costs are always integer numbers of cents)?

30. x is a real number satisfying $2^x * 4^{2x} * 8^{3x} * 16^{4x} = 32^{18}$. Compute x.

31. How many ways can 3 boys and 4 girls sit in a line so that no one sits next to another person of the same gender?

32. A regular polygon is inscribed in a circle. The circle has an area of 16π units and each side of the polygon has a length of 4 units. How many sides does the inscribed polygon have?

33. A spinner is divided into 4 equal sections, and 2 non-adjacent ones are painted black. Alex the Kat then divides every white section into 3 equal regions, and paints the middle one black. Kelvin the Frog then divides every black region, including the ones formed by Alex the Kat, into 3 equal regions and paints the middle one white. Alex the Kat then spins the spinner, winning if it lands on a black region. What is the probability that he wins?

34. Three unit circles are each externally tangent to each other. Find the area of the smallest equilateral triangle that contains each of these three circles.

35. Find the product of all values of x satisfying $(x+4)^{3x+1} = 1$.

36. Triangle ABC has lengths $AB = 6, BC = 8, CA = 10$. Triangle ABD has lengths $AD = BD = 5$. Compute the area of quadrilateral $ADBC$.

37. Point P lies in square $ABCD$ such that ABP is an equilateral triangle. Find the measure, in degrees, of angle CDP.

38. Let $a_0 = 1$ and $a_n = \sqrt{(n+2)a_{n-1} + 1}$ for $n \geq 1$. Find a_{2014}.

39. How many of the numbers from 1 to 2014 have a 6 in it?

40. Let $T_n = 1 + 2 + \cdots + n$. Compute the value of k for which $T_{k^2} = 666$.

41. A right triangle has one side length of length 15, and one of the other two side lengths is twice the other. Find the product of all possible values for the smallest side length.

42. Let $f(x) = x^2 - 6x + 5$ and $g(x) = x^2 - 7x + 12$. Find the sum of all x satisfying the equation $(f(x)+g(x))^2 - (f(x) - g(x))^2 = 0$.

43. ABC is a right triangle with right angle B, and regular hexagons P_1, P_2, P_3 are constructed outside ABC such that AB is a side of P_1, BC is a side of P_2, and P_3 is a side of CA. If the area of P_1 is 16 and the area of P_2 is 36, find the area of P_3.

44. Suppose $f(x)$ is a function such that $f(x) + x \cdot f(3-x) = 2$. Compute $f(1)$.

45. A point is randomly chosen in a square with side length 2. 4 line segments are drawn, one from each vertex to the point, forming 4 triangles. What is the probability that the sum of the areas of some two of these triangles is less than $\frac{2}{3}$?

46. Alex the Kat gives Kelvin the Frog three positive integers $a, b,$ and c, and challenges Kelvin to find the value of $a^b - a^c$. Kelvin, being a frog, instead finds the value of a^{b-c}. Incredibly, Kelvin is still correct! Find $a + b + c$.

47. Find the number of integers between 1 and 20, inclusive, that can be written as the sum of two squares of integers.

48. $ABCD$ is a square with side length one. Four circles of radius one are drawn, centered at A, B, C, and D respectively. Find the area of the intersection of these four circles.

49. ABC is a right triangle with right angle $B, AB = 10$, and $BC = 24$. Let M be the midpoint of AC. Circle O_1 is inscribed in triangle ABM, and circle O_2 is inscribed in triangle BCM. Let r_1 be the radius of O_1, and let r_2 be the radius of O_2. Find the value of $r_1 r_2$.

50. The two digit integers $\overline{ab}$ and $\overline{bc}$, where a, b, and c are distinct digits, are to be divided and the result is to be simplified. Kelvin the Frog accidentally cancels the b's, resulting in an answer of $\frac{a}{c}$. Remarkably, his answer is still correct! Find the sum of all possible values of $\overline{ab}$ such that this is the case. Note that $\frac{a}{c}$ is not necessarily the most simplified version of the fraction.