

1. There are 16 words in the sentence, and exactly 5 of them have four letters, as shown: “**What** is the probability **that** a randomly chosen **word** of **this** sentence has exactly **four** letters?”

Therefore, the desired probability is simply $\boxed{\frac{5}{16}}$.

2. Notice that $9 \cdot 18 = 162$ and $9 \cdot 19 = 171$. Any other multiple of 9 will be further away from 169, so our answer is the closer of the two - $\boxed{171}$.
3. Applying the definition, we have $2\#0 = 2 \cdot 0 - 2 - 3 = -5$ and $1\#4 = 1 \cdot 4 - 1 - 3 = 0$. Thus,

$$(2\#0)\#(1\#4) = 5\#0 = (-5) \cdot 0 - (-5) - 3 = \boxed{2}.$$

4. Since five students take both languages, $27-5=22$ students take French only and $32-5=27$ students take Spanish only. Hence a total of $22 + 27 = \boxed{49}$ students take exactly 1 language course.
5. Except for the middle letter, every palindrome must contain an even number of every present letter. Suppose the letter “b” is the middle letter. Then there must be at least 2 g’s, at least 2 t’s, at least 2 r’s, and at least 2 o’s. Hence there are at least $1 + 2 + 2 + 2 + 2 = \boxed{9}$ letters in such a palindrome. One example is the palindrome “rtgobogtr”.
6. If Alex writes x more problems, then he will have written a total of $61 + x$ of the $187 + x$ problems. Hence

$$\begin{aligned} \frac{61 + x}{187 + x} &= \frac{1}{2} \\ \implies 122 + 2x &= 187 + x \\ \implies x &= \boxed{65} \end{aligned}$$

7. We have $0^2 = 0, 1^2 = 1, 2^2 = 4, 3^2 = 9, 4^2 = 16, 5^2 = 25, 6^2 = 36, 7^2 = 49, 8^2 = 64,$ and $9^2 = 81$. Any other perfect squares will have the same last digit as one of these, so the possible last digits are 0, 1, 4, 5, 6, and 9. There are $\boxed{6}$ of these.
8. After 5 days, the hedgehog has made $5 \cdot 5 = 25$ new friends, for a total of $4+25 = \boxed{29}$. Alternatively, in general the hedgehog will have $5(n-1)+4 = 5n - 1$ friends on day n by the same logic, so when $n = 6$ he will have 29 friends.

9. Notice that

$$\begin{aligned} \frac{(6!)^2}{5!7!} &= \frac{6!}{5!} \cdot \frac{6!}{7!} \\ &= \frac{6 \cdot \cancel{5} \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}}{\cancel{5} \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}} \cdot \frac{\cancel{6} \cdot \cancel{5} \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}}{7 \cdot \cancel{6} \cdot \cancel{5} \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}} \end{aligned}$$

$$= \boxed{\frac{6}{7}}$$

10. We can prime factorize $12 = 2^2 \cdot 3$ and $18 = 2 \cdot 3^2$. Hence, since the greatest common divisor takes the smallest exponent of each prime, we have $\gcd(12, 18) = 2 \cdot 3 = 6$. Similarly, since the least common multiple takes the largest exponent of each prime, we have $\text{lcm}(12, 18) = 2^2 \cdot 3^2 = 36$. Thus our answer is $6 + 36 = \boxed{42}$.

Alternatively, we can utilize the Euclidean Algorithm to find $\gcd(12, 18) = \gcd(12, 6) = \gcd(6, 6) = 6$, then use $\text{lcm}(x, y) = \frac{xy}{\gcd(x, y)}$ to find $\text{lcm}(12, 18) = \frac{12 \cdot 18}{6} = 36$. Our answer is then the same as before.

11. In general, the number exactly halfway between a and b is $\frac{a+b}{2}$ - the average of the two numbers. Therefore, the number halfway between $\frac{1}{6}$ and $\frac{1}{4}$ is:

$$\frac{\frac{1}{6} + \frac{1}{4}}{2} = \frac{\frac{2}{12} + \frac{3}{12}}{2} = \frac{\frac{5}{12}}{2} = \boxed{\frac{5}{24}}$$

12. Let his final exam score be x . Then

$$\begin{aligned} \frac{91 + 89 + 88 + 94 + 87 + 85 + x}{7} &= x \\ \implies \frac{534 + x}{7} &= x \\ \implies 534 + x = 7x &\implies 534 = 6x \\ \implies x &= \boxed{89} \end{aligned}$$

13. We have $2 \star 2 = 2^2 + 2^2 = 4 + 4 = 8$, and thus $2 \star (2 \star 2) = 2 \star 8 = 2^8 + 8^2 = 256 + 64 = \boxed{320}$.

14. Let the integers be $x - 5, x - 4, \dots, x + 4, x + 5$. Then we have $11x = 1331 \implies x = 121$, making the largest of them $x + 5 = 121 + 5 = \boxed{126}$.

15. To mix the color purple, Rita would have to roll red once and blue once. She can either roll red first and blue next, which has a probability of $\frac{3}{6} \cdot \frac{1}{6} = \frac{1}{12}$, or roll blue first and red next, which also has a probability of $\frac{1}{6} \cdot \frac{3}{6} = \frac{1}{12}$. The final probability is thus $\frac{1}{12} + \frac{1}{12} = \boxed{\frac{1}{6}}$.

16. Let a and b be the integers, and without loss of generality assume that $a \geq b$ (if $a < b$, we can simply switch the two). Since $8 = a + b \leq 2a$, we know that a is at least 4. Furthermore, since $a^2 \leq a^2 + b^2 = 34$, we know that a is less than 6. Therefore we only need to check $a = 4$ and $a = 5$, the latter of which leads to the solution $a = 5, b = 3$. Therefore, $ab = 5 \cdot 3 = \boxed{15}$.
17. Cross-multiplying, the equation is equivalent to $(x^2 + 1)(x + 1) = (x^2 - 1)(x - 1)$. Expanding gives $x^3 + x^2 + x + 1 = x^3 - x^2 - x + 1 \implies 2(x^2 + x) = 0 \implies x^2 + x = 0 \implies x(x + 1) = 0$. Thus $x = 0$ or $x = -1$. However, if $x = -1$, then the fraction $\frac{x^2 - 1}{x + 1}$ will have denominator zero, meaning that the only valid solution is $x = \boxed{0}$.
18. After the six-year period, the first stock is equal to $1.3 \cdot .7 \cdot 1.3 \cdot .7 \cdot 1.3 \cdot .7 = (.91)^3$ of its original value. This is equivalent to it being worth 91% of its previous value for each of three years, which is a loss of $\boxed{9}\%$.
19. It takes Lev 20 minutes to reach 100, at which point Alex wakes up and begins running. Every minute, Alex runs 2 more integers than Lev does, so it will take 50 minutes for Alex to catch up. Thus, the total time before Alex catches up to Lev is $20 + 50 = \boxed{70}$ minutes.
20. Note that the surface area varies with the square of the side length, and the volume varies with the cube of the side length. In other words, multiplying the side length by x is the same as multiplying the surface area by x^2 and the volume by x^3 . In this case, we are adding 20% to the side length, equivalent to multiplying by 1.2. Therefore, the surface area of the cube is increased by a factor of 1.44, or 44%, and the volume of the cube is increased by a factor of 1.728, or 72.8%. Thus $x = 44$ and $y = 72.8$, meaning $5(y - x) = 5(72.8 - 44) = \boxed{144}$.
21. There are two cases to consider. The first is when Alex wins on his first turn, and the second is when Alex wins on his second turn. Notice that Alex will never get a third turn, as by then all the spots will necessarily be winning. The first case necessitates Kelvin not rolling an even number on his first turn, which occurs with a probability of $\frac{1}{2}$, and Alex rolling either an even number or the number Kelvin rolled, which occurs with a probability of $\frac{2}{3}$. Hence the probability of this case occurring is $\frac{1}{3}$. The second case requires Kelvin rolling an odd number, followed by Alex not winning, followed by Kelvin rolling the only number remaining that isn't winning. Then all 6 numbers will be winning, and Alex will win. The probability of this case is $\frac{1}{2} \cdot \frac{1}{3} \cdot \frac{1}{6} = \frac{1}{36}$, making the total probability $\frac{1}{3} + \frac{1}{36} = \boxed{\frac{13}{36}}$.

22. Notice that $4^x - 1 = (2^x)^2 - 1 = (2^x - 1)(2^x + 1)$, meaning that $4^x - 1$ can be prime only if $2^x - 1 = 1$ or $2^x + 1 = 1$. The latter is impossible as there is no positive integer x such that $2^x = 0$, while the former leads to $2^x = 2 \implies x = 1$. This is the only such value, and thus our answer is $\boxed{1}$.
- Alternatively, notice that 4^x always leaves a remainder of 1 upon division by 3, and thus $4^x - 1$ is always divisible by 3. As such, the only way $4^x - 1$ can be prime is if $4^x - 1 = 3 \implies 4^x = 4 \implies x = 1$, and the rest is as before.
23. Since the triangle is isosceles, some two of its side lengths are equal. Hence either $x - 4 = 2x - 9 \implies x = 5$, $2x - 9 = 3x - 15 \implies x = 6$, or $x - 4 = 3x - 15 \implies x = \frac{11}{2}$. However, if $x = 5$, the side lengths of the triangle are 1, 1, and 0 - clearly not a triangle. Hence the only two valid values of x are 6 and $\frac{11}{2}$, the sum of which is $\boxed{\frac{23}{2}}$.
24. Every two days, Steven's name increases by 6 words. Hence, on day 15 (which is 14 days after the first), Steven's name is 42 words. One day following that, Steven's name increases by two words, leading to his 44-word name. This occurs on day $\boxed{16}$.
25. Since the triangle's altitudes are all equal, the triangle must be equilateral. Suppose the side length of this triangle is $2s$. Then, by the Pythagorean theorem, $s^2 + 6^2 = (2s)^2 \implies 36 = 3s^2 \implies 12 = s^2$. The area of the triangle is thus $\frac{s^2\sqrt{3}}{4} = \frac{12\sqrt{3}}{4} = \boxed{3\sqrt{3}}$.
26. To get at least 21 points on the USAMO, Alex will need to expend at least 210 units of effort. To get at least 200 points on the AMC and AIME combined, Alex can expend 102 units of effort by focusing on the AMC alone, or $96+7=103$ units of effort by getting 192 points on the AMC and 10 on the AIME. Any lower amount of effort on the AMC is clearly counterproductive, as he gets a higher rate of points to effort on the AMC than on the AIME. Hence the minimum possible amount of effort he can expend is $210 + 102 = \boxed{312}$.
27. Let the number be \overline{abc} . If any of a, b, c are zero, then the product of the digits would be zero, implying that the sum of the digits is 0 - impossible. Hence a, b, c are all positive digits. WLOG assume that $a \leq b \leq c$ - we'll account for the permutations later. Thus we have $abc = a + b + c \leq 3c \implies ab \leq 3$, meaning we need check only $(a, b) = (1, 1)$, $(a, b) = (1, 2)$, and $(a, b) = (1, 3)$. The former case gives us $c = 2 + c$, contradiction, the second gives us $2c = 3 + c \implies c = 3$, and the third gives us $3c = 4 + c \implies c = 2$, a contradiction as we assumed $b \leq c$. Therefore the only possible numbers are the permutations of $\overline{123}$, of which there are $3! = \boxed{6}$.

28. Let X and Y be the altitudes from E to AB and G to CD respectively. Then, by the Pythagorean theorem, $EX = GY = \frac{\sqrt{3}}{2}$. Hence $EG = \sqrt{3} + 1$. Therefore, the area of $EFGH$ is

$$\frac{EG^2}{2} = \frac{(\sqrt{3} + 1)^2}{2} = \boxed{2 + \sqrt{3}}.$$

29. Let the bill be x . Then $\lceil x \rceil - x > .2x \implies \lceil x \rceil > 1.2x$. Since $\lceil x \rceil < x + 1$, this tells us that $1.2x < x + 1 \implies .2x < 1 \implies x < 5$, hence the largest possible cost will occur when $4 < x < 5$, or $\lceil x \rceil = 5$. Then $5 > 1.2x \implies \frac{25}{6} > x \implies 4.1\bar{6} > x$, making the largest possible cost $\boxed{416}$ cents.

30. Note that

$$\begin{aligned} 2^x \cdot 4^{2x} \cdot 8^{3x} \cdot 16^{4x} &= 2^x \cdot (2^2)^{2x} \cdot (2^3)^{3x} \cdot (2^4)^{4x} \\ &= 2^x \cdot 2^{4x} \cdot 2^{9x} \cdot 2^{16x} = 2^{30x} = (2^5)^{18} = 2^{90} \\ &\implies 30x = 90 \implies x = \boxed{3}. \end{aligned}$$

31. The only way that this is possible is if the children alternate in the form “GBGBGBG”, where “G” denotes a girl and “B” denotes a boy. The 4 girls can arrange themselves in any of $4! = 4 \cdot 3 \cdot 2 \cdot 1 = 24$ ways, while the 3 boys can arrange themselves in $3! = 3 \cdot 2 \cdot 1 = 6$ ways. Thus there are a total of $6 \cdot 24 = \boxed{144}$ possible arrangements.

32. Let O be the center of the circle, and A, B be two adjacent vertices of the polygon. Since the area of the circle is 16π , its radius is 4, and hence $OA = OB = AB = 4$. Therefore OAB is equilateral, and $\angle AOB = 60^\circ$. There are thus $\frac{360^\circ}{60^\circ} = \boxed{6}$ sides to the polygon.

33. Before any of the operations, $\frac{1}{2}$ of the spinner is black and $\frac{1}{2}$ is white. Alex then changes $\frac{1}{3}$ of the white region, or $\frac{1}{6}$ of the whole spinner, to black, making $\frac{2}{3}$ of the spinner black and $\frac{1}{3}$ of it white. Kelvin then changes $\frac{1}{3}$ of the black region, or $\frac{2}{9}$ of the whole spinner, to white, making $\frac{4}{9}$ of the spinner black and $\frac{5}{9}$ of it white. Our answer is therefore $\boxed{\frac{4}{9}}$.

34. Let O_1, O_2 , and O_3 be the centers of the three unit circles, and ABC be the desired equilateral triangle. Dropping altitudes from O_1 and O_2 to \overline{AB} , meeting the line at C and D respectively, gives $AB = AC + CD + DB$. Since $O_1C = 1$ and $\angle ABO_2 = \angle BAO_1 = 30^\circ$, we have $AC = DB = \sqrt{3}$. Furthermore, $CD = O_1O_2 = 2$. Thus $AB = 2 + 2\sqrt{3}$, making the area of ABC equal to $\frac{s^2\sqrt{3}}{4} = (\sqrt{3} + 1)^2\sqrt{3} = \boxed{6 + 4\sqrt{3}}$.

35. There are three ways this is possible. Either $x + 4 = 1$, in which case $(x + 4)^{3x+1} = 1$ as $1^n = 1$ for any n , or $3x + 1 = 0, x + 4 \neq 0$ as $n^0 = 1$ for any $n \neq 0$, or $x + 4 = -1$ and $3x + 1$ is even, as $(-1)^n = 1$ for any even n . These lead to the three valid solutions $x = -3, x = -\frac{1}{3}, x = -5$, the product of which is $\boxed{-5}$.
36. Since $AB^2 + BC^2 = 6^2 + 8^2 = 36 + 64 = 100 = 10^2 = CA^2$, ABC is a right triangle. Hence the area of this triangle is $\frac{6 \cdot 8}{2} = 24$. Let E be the foot of the altitude from D to AB , so that $AE = BE = \frac{6}{2} = 3$. Then $DE = \sqrt{5^2 - 3^2} = \sqrt{16} = 4$, hence the area of triangle ABD is $\frac{6 \cdot 4}{2} = 12$. Thus, the area of quadrilateral $ADBC$ is $24 + 12 = \boxed{36}$.
37. Since $\angle ABP = 60^\circ$, and $PB = AB = BC$ means that $\triangle PBC$ is isosceles, we have $\angle PBC = 90^\circ - 60^\circ = 30^\circ \implies \angle PCB = 75^\circ$. Then $\angle DCP = \angle CDP = \boxed{15^\circ}$.
38. Notice that $a_0 = 1, a_1 = \sqrt{3 \cdot 1 + 1} = 2, a_2 = \sqrt{4 \cdot 2 + 1} = 3$, and so on. Thus I claim that $a_n = n + 1$. Since $n + 1 = \sqrt{(n + 2) \cdot n + 1} = \sqrt{(n + 2) \cdot a_{n-1} + 1}$, this is true. Therefore $a_{2014} = 2014 + 1 = \boxed{2015}$.
39. We will instead count the numbers from 1 to 2014 that do *not* have a 6 in it. There are $2 \cdot 9 \cdot 9 \cdot 9 - 1 =$ such numbers from 1 to 1999, as there are two choices (0 or 1) for the thousands digit and nine for each of the hundreds, tens, and ones digit (excluding the case of 0000), so there are $1999 - 1457 = 542$ numbers between 1 and 1999 that do have a 6 in it. Adding in the final case of 2006, there are $\boxed{543}$ such numbers between 1 and 2014.
40. Note that $T_n = \frac{n(n+1)}{2}$, hence $T_{k^2} = \frac{k^2(k^2+1)}{2} = 666$. Since $666 \cdot 2 = 2^2 \cdot 3^2 \cdot 37$, we have $666 = \frac{36 \cdot 37}{2} \implies k^2 = 36 \implies k = \boxed{6}$. We also accepted $k = -6$.
41. If 15 is one of the legs, then we have $15^2 + x^2 = (2x)^2 \implies 15^2 = 3x^2 \implies x = \frac{15}{\sqrt{3}} = 5\sqrt{3}$ where x is the length of the other leg. If 15 is the hypotenuse, then we have $x^2 + (2x)^2 = 15^2 \implies 5x^2 = 15^2 \implies x = \frac{15}{\sqrt{5}} = 3\sqrt{5}$, where x is the length of the shorter leg. The product of these is $\boxed{15\sqrt{15}}$.
42. Expanding the equation gives

$$f(x)^2 + 2f(x)g(x) + g(x)^2 - f(x)^2 + 2f(x)g(x) - g(x)^2 = 0$$

so $4f(x)g(x) = 0 \implies f(x)g(x) = 0$. Thus either $f(x) = 0$ or $g(x) = 0$. The solutions to the former are $x = 1$ and $x = 5$, while the solutions to the latter are $x = 3$ and $x = 4$. Their sum is $\boxed{13}$. Alternatively, Vieta's formulas tell us that the sum of the 4 roots is $6 + 7 = \boxed{13}$.

43. Notice that the fact that the constructed figures are hexagons is irrelevant, as it is only relevant that the area is directly proportional to the square of the side length - true for any similar polygons. Hence, as long as we are constructing similar shapes on each side, it does not matter which shape we choose. Thus we can assume instead that they are squares, and then the Pythagorean theorem immediately tells us that $P_3 = P_1 + P_2 = 16 + 36 = 52$.

Alternatively, the area of a regular hexagon with side length s is $\frac{s^2\sqrt{3}}{4}$, so we can solve for $AB^2 = \frac{64}{\sqrt{3}}$ and $BC^2 = 48\sqrt{3}$, hence $CA^2 = \frac{208\sqrt{3}}{3}$ and $\frac{CA^2\sqrt{3}}{4} = \boxed{52}$.

44. Plugging in $x = 1$ and $x = 2$ gives us $f(1) + 1 \cdot f(2) = 2$ and $f(2) + 2 \cdot f(1) = 2$, hence $f(1) + f(2) = f(2) + 2f(1) \implies f(1) = \boxed{0}$.

45. Suppose that the points of the square lie at $(0, 0)$, $(2, 0)$, $(2, 2)$, and $(0, 2)$. 2 of the 4 triangles that don't share a side have areas summing to 2, so the two triangles in question must share a side. WLOG assume that the point is in the bottom-left quarter of the square, i.e. $0 \leq x, y \leq 1$. We will multiply our answer by 4 to account for the other possibilities later. Then the areas of the two smallest triangles are $\frac{x \cdot 2}{2} = x$ and $\frac{y \cdot 2}{2} = y$, so $x + y \leq \frac{2}{3}$. This region is a right triangle with legs $\frac{2}{3}$ and $\frac{2}{3}$, and thus has an area of $\frac{2}{9}$. Multiplying this by 4 to account for the other cases, the

area of the desired region is $\frac{8}{9}$. Our answer is thus $\frac{8}{2^2} = \boxed{\frac{2}{9}}$.

46. Since a, b, c are all positive, a^{b-c} is also positive and hence $b > c$. Furthermore, division by a^c gives us $a^{b-c} - 1 = a^{b-2c}$. Since $b > c$, a^{b-c} is an integer, and hence the LHS is an integer. Therefore, so is a^{b-2c} , implying that $b \geq 2c$. If $b - 2c > 0$, then $a \mid a^{b-c} - a^{b-2c} = 1$, implying that $a = 1$ - a clear contradiction as $1 - 1 \neq 1$. Hence $b - 2c = 0$, and thus $a^{b-2c} = 1$. Thus $a^{b-c} - 1 = 1 \implies a^{b-c} = 2$, possible only if $a = 2$ and $b - c = 1$. Solving gives us $b = 2, c = 1$, which is indeed a solution as $2^2 - 2^1 = 2^{2-1}$. Hence $a + b + c = \boxed{5}$.

47. The squares less than 20 are $0^2 = 0, 1^2 = 1, 2^2 = 4, 3^2 = 9$, and $4^2 = 16$. There are $\binom{5}{2} + 5 = 15$ combinations of these, but $16 + 16$ and $9 + 16 > 20$ and $0 + 0 < 1$. Hence $15 - 3 = \boxed{12}$ of these lie between 1 and 20. Notice that it is also easily verifiable that no combinations result in the same sum.

48. Let E, F, G, H be the intersections of the circles, so that ABE, BCF, CDG, DAH are equilateral. Then the distance from G to AB is $1 - \frac{\sqrt{3}}{2}$, as the altitude from G to CD is $\sqrt{3}2$ by the Pythagorean theorem, and so is the distance from E to CD . Hence $EG = \sqrt{3} - 1$. Therefore $GH = \frac{\sqrt{6} - \sqrt{2}}{2}$, and as such the altitude from D to GH has length $\frac{\sqrt{6} + \sqrt{2}}{4}$. This makes the area

of $DGH = \frac{1}{4}$, hence the area formed by \overline{GH} and arc GH is $\frac{\pi}{12} - \frac{1}{4}$. As such, our final answer is

$$\left(\frac{\sqrt{6} - \sqrt{2}}{2}\right)^2 + 4\left(\frac{\pi}{12} - \frac{1}{4}\right) = \boxed{\frac{\pi}{3} + 1 - \sqrt{3}}.$$

49. Since $\angle ABC$ is right, B lies on the circle with diameter AC , and therefore $MA = MB = MC = 13$. Thus

$$r_1 \cdot \frac{13 + 13 + 10}{2} = [ABM] = \frac{1}{2}[ABC] = 60$$

and

$$r_2 \cdot \frac{13 + 13 + 24}{2} = [ACM] = \frac{1}{2}[ABC] = 60$$

$$\text{Hence } r_1 r_2 = \frac{120^2}{36 \cdot 50} = \boxed{8}.$$

50. We have

$$\begin{aligned} \frac{10a + b}{10b + c} = \frac{a}{c} &\implies 10ac + bc = 10a + ac \\ \implies 9ac + bc = 10ab &\implies 9ac = b(10a - c) \end{aligned}$$

If $9 \mid 10a - c$, then since $a \neq c$ the only possibility is $a = 9, c = 0$, but this implies $b = 0$ which is a contradiction. Hence $9 \nmid 10a - c$, and thus $3 \mid b$. If $b = 3$ then $c = \frac{10a}{3a+1}$, and since $\gcd(a, 3a+1) = 1$ this implies that $3a+1 \mid 10$. That is possible only if $a = 3$, but then $a = b$ - contradiction. Hence $b \neq 3$. If $b = 6$, then $c = \frac{20a}{3a+2}$. If a is odd then $\gcd(a, 3a+2) = 1$, implying that $3a+2 \mid 20$, possible only when $a = 1$. This leads to $\overline{ab} = 16$, true as $\frac{16}{64} = \frac{16}{64} = \frac{1}{4}$. If a is even then $3a+2 \mid 40$, possible only when $a = 2$. That leads to $\overline{ab} = 26$, true as $\frac{26}{65} = \frac{26}{65} = \frac{2}{5}$. Finally, when $b = 9$ we have $c = \frac{10a}{a+1}$, possible only when $a+1 \mid 10 \implies a = 1, 4$. Both of these are valid as $\frac{19}{95} = \frac{19}{95} = \frac{1}{5}$ and $\frac{49}{98} = \frac{49}{98} = \frac{4}{8}$, so our answer is $49 + 19 + 26 + 16 = \boxed{110}$.