2015 Joe Holbrook Memorial Math Competition
6th Grade Exam

The Bergen County Academies Math Team

October 11th, 2015

Instructions

DO NOT TURN OVER THIS PAGE UNTIL YOU ARE TOLD TO DO SO.

The Joe Holbrook Memorial Math Competition is a 90-minute, 50-question exam. Each question has exactly one correct answer; writing two different answers, EVEN IF ONE IS CORRECT, is worth no credit. In addition, **only answers written on the answer sheet provided will be graded.**

Be advised that proctors cannot answer any questions about terminology, notation or the questions.

On the JHMMC, you may only use writing utensils, erasers, and scrap paper provided by the proctors. You MAY NOT use calculators, compasses, protractors, straightedges, or your own scrap paper.

All answers must be fully simplified and exact. For example:

- Write $2\sqrt{2}$, rather than $2.83\ldots$
- Write $\frac{4}{3}$, rather than $1\frac{1}{3}$ or $1.33\ldots$

In addition, know that $[A_1A_2\cdots A_n]$ denotes the area of n-gon $A_1A_2\cdots A_n$.

1. What is $7 + 9 + 10 + 10 + 7 + 2 + 2 + 1 + 1 + 1$?

2. Find $3 \times 5 + 4 \times 2$.

3. Jen loves multiplying numbers. Every time you tell her two numbers, she multiplies them and shouts out the answer. I told Jen the numbers five and six. What number does she shout out?

4. How many times does the Sun set between noon on Saturday and noon on the following Thursday?

5. Nemo has 11 goldfish in his fish tank. Each goldfish needs to eat 19 goldfish flakes daily. How many goldfish flakes, in total, does Nemo need each day to be able to feed all 11 goldfish?

6. What is the remainder when 123456 is divided by 5?

7. Find the 100th member of the following sequence: $1, 4, 7, 10, \ldots$

8. Compute $3(3(3+2)+2(3+2))+2(3(3+2)+2(3+2))$.

9. How many positive integers less than 100 are divisible by 7?

10. The mean of 2015 consecutive integers is 2. What is the median?

11. One floop is equal to 4 bleeps. 8 bleeps is equal to 3 plops. How many floops is equal to 48 plops?

12. Andrew started with seventeen pieces of candy and Jackson started with fourteen pieces of candy. Andrew went to the candy store and bought three more pieces of candy. Jackson then ate two pieces of candy. Finally, Andrew gave 5 pieces of candy to Jackson. What is the positive difference between the number of pieces of candy Andrew has and the number of pieces of candy Jackson has?

13. Thomas the Mouse wants to eat cheese and crackers. He has 3 kinds of cheese and 4 kinds of crackers. How many ways are there to choose 1 cheese and 1 cracker?

14. What is the smallest positive integer divisible by 4, 5, and 6?

15. What is $0 - 1 + 2 - 3 + 4 - 5 + \cdots - 99 + 100$?

16. What is $\frac{1}{2} + \frac{2}{3} + \frac{3}{4}$?

17. Jungle Jim, the proprietor of the jungle gym, is charging admission. If the admission fee for Matt is \$20, but he has a coupon for a 10% discount, and the admission fee for Tanny is \$25, but he has a coupon for a 15% discount on every dollar after \$5 that he pays, who pays more, and how much does he pay?

18. Alex the Kat is chasing Thomas the Mouse, who has a 20 foot lead. Mice run at 1 foot every two seconds, whereas Alex the Kat runs at 1 foot every one second. How many seconds does it take for Alex the Kat to catch Thomas the Mouse?

19. What is the last digit of 2^{245}?

20. Let $s(n)$ be the sum of the digits of an integer n. Compute $s(111111^2)$.

21. Esther rolls two fair dice each with faces numbered 1 through 6. What is the probability that the product of the two rolled numbers is odd?

22. Compute $(23 + 46 \times 23 + \frac{46}{2}) - 48 \times 23$.

23. How many multiples of 17 are three-digit positive integers?

24. Young Guy is selecting toppings for ice cream. There are 3 options: sprinkles, whipped cream, and fudge. If he is allowed to select any number of toppings, or no toppings at all, how many different ways can he have ice cream?

25. A square and a regular hexagon have the same perimeter. The square has side length 18. What is the side length of the hexagon?

26. If 75 Puzzles = 10 Dragons and 2 Dragons = 5 YoungGuys, then 1 YoungGuy is how many Puzzles?

27. How many distinct arrangements are there of the letters JHMMC?

28. Kelvin the Frog has two six-sided die. One is labeled $1, 2, 2, 3, 3, 4$, and the other is labeled $1, 3, 4, 5, 6, 8$. What is the probability that he gets a sum of 5 when he rolls the die?

29. In Part A of his math test, Minsung got seven out of ten questions correct. If there are fifteen questions in Part B of the test and all the questions are worth the same number of points, what is the least number of questions on Part B that Minsung must answer correctly to get a final score of at least 80%?

30. Find the sum $10 + 11 + \cdots + 100$.

31. What is the probability that a random positive two digit integer is greater than or equal to the number resulting from switching its digits?

32. Compute the largest prime factor of $6^{12} + 12^6$.

33. Define a sequence of integers as follows: $a_0 = 0, a_1 = 1, a_2 = 2$ and for $n \geq 3$, $a_n = a_0 + \cdots + a_{n-1}$. Find the value of a_8.

34. What is the largest n such that $1^1 + 2^2 + \cdots + n^n \leq 50000$?

35. What is the side length of a cube whose volume is numerically twice its surface area?

36. Consider the expression A X B Y C, where (A,B,C) is some arrangement of $(0, 1, 2)$ and (X,Y) is some arrangement of $(+, \times)$. (For example, it could represent $2 + 0 \times 1$.) How many different values, respecting order of operations, can the expression A X B Y C take?

37. In a baseball game, the probability that Uncle Boris will pitch with his right hand is $\frac{5}{8}$ and the probability he throws with his left hand is $\frac{3}{8}$. He has a $\frac{4}{5}$ chance of throwing a good pitch with his right hand but only a $\frac{4}{9}$ chance of a good pitch with his left. What is the probability that Uncle Boris will make a good pitch in any given throw?

38. Find the difference between the degree measure of an internal angle of a regular octagon and an external angle of a regular decagon.

39. If the sum of the digits of a 3-digit number is 15, and the number itself is a multiple of 35, what is this number?

40. **The Stangulator** creates a list of digits in the following way: 1 is listed 1 time, followed by 2 being listed 2 times, followed by 3 being listed 3 times, etc., and in general, n being listed n times. What digit occupies the 2015th position on **The Stangulator**'s list?

41. A coin lands on heads with likelihood $\frac{2}{3}$ and tails with likelihood $\frac{1}{3}$. If Ryan the ⋔ tosses the coin 4 times, what is the probability that Ryan the ⋔ will see at least two heads?

42. There is a sequence of positive real numbers $a_1, a_2, a_3, \ldots$ such that for $n \geq 2$, each term a_n is the product of all the previous terms. If $a_7 = 41$, what is a_8?

43. Two regular pentagons have areas 8 and 18 square meters. If the perimeter of the smaller pentagon is 6, what is the perimeter of the larger pentagon in meters?

44. Three faces of a rectangular prism have areas 16, 20, and 45. What is the volume of this prism?

45. Compute the smallest positive integer with at least 20 positive integer factors.

46. Let a, b, c, d be integers, and let $b > 0$. If $a + b = c$, $b + c = d$, and $c + d = a$, what is the maximum possible value $a + b + c + d$ can take?

47. AJ the Dennis rolls 3 fair 6-sided dice. He notices that his largest number rolled is exactly twice his smallest number. What is the probability that this happens?

48. Suppose N is the number of perfect squares that divide 2015^{2015}. How many perfect squares divide N?

49. Rebecca can only move parallel to the x-axis (at a constant speed of 20 mph) and parallel to the y-axis (at a constant speed of 10 mph), while The Great Bustard can move in any direction at 10 mph. If both Rebecca and The Great Bustard start at the origin, what is the area of the region $|x| < 1$ and $|y| < 1$ in the xy plane that The Great Bustard can reach before Rebecca?

50. In acute triangle ABC, $AB = 4$ and $AC = 9$. Let M be the midpoint of BC in $\triangle ABC$, and let ω be a circle with center O that is tangent to BC at point C with the property that BO is perpendicular to AM. Let ω intersect AC at point N. What is $|AN - NC|$?