2015 Joe Holbrook Memorial Math Competition
8th Grade Exam Solutions

The Bergen County Academies Math Team

October 11th, 2015

1. $1 + 3 + 6 + 8 + 10 + 8 + 6 + 4 + 2 + 2 = \boxed{50}$.

2. Sixteen years ago, Sunshine was four years old, while Moonbeam was not yet born, so $\boxed{\text{Moonbeam}}$ was born second.

3. There are three ways to choose the cheese. Once Thomas has chosen the cheese, he now has four options for his cracker. Thus for each of the three cheeses, there are four cracker options, so the number of combinations is $3 \times 4 = \boxed{12}$.

4. There are 2 hours and 29 minutes from 5:00 to 7:29. Each hour is 60 minutes long, so this is $2(60) + 29 = 149$ minutes. Each episode is 20 minutes long, so the number of full episodes that cam be watched is the integer part of $\frac{149}{20}$, which is $\boxed{7}$.

5. Recognizing that each number is one more than the one before it, we can group the terms into pairs:

   $0 + (-1 + 2) + (-3 + 4) + \cdots + (-99 + 100) = 0 + 1 + 1 + \cdots + 1$. Each positive even number ends a pair, and there are 50 positive even integers less than 100, so the final sum is $\boxed{50}$

6. The *triangle inequality* states that the sum of the lengths any two sides of a triangle are greater than the third. Think about it: if the sum was less than the longest side, then you couldn't close the triangle! Knowing this, we know that the longest length of the third side has to be less than $12 + 13 = 25$; the greatest such integer is $\boxed{24}$.

7. First, we compute what Matt pays. His discount is $10\% \times \$20 = 0.1 \times \$20 = \$2$, so he pays \$18 total. Tanny pays $\$25 = \$20 + \$5$, so his 15% discount applies only to \$20: $15\% \times \$20 = 0.15 \times \$20 = \$3$ is his discount, so he pays in total \$22. Thus, the answer is $\boxed{\text{Tanny, \$22}}$.

8. To get the least product, we want to end with a big, negative number. To achieve this, we pick two big factors with opposite signs; $-7 \times 3 = \boxed{-21}$.

9. Let x be the favorite number. Rewriting the question as a mathematical equation, we have that $3(x + 9) = p^4$, where p is the smallest odd prime number; this is 3. So, $3(x + 9) = 3^4 = 81 \implies 3x + 27 = 81 \implies 3x = 54 \implies x = 18$. We are looking for half of x, which is $\boxed{9}$.

10. If the product of two integers is odd, then both of them must have been odd. This means that the probability of the product being odd is the same as the probability of having rolled two odd numbers. The probability of rolling an odd number for a single roll is $\frac{1}{2}$, and dice rolls are independent events, so we can compute the probability by multiplying: $\frac{1}{2} \times \frac{1}{2} = \boxed{\frac{1}{4}}$.

11. An isosceles triangle has two distinct side lengths; that of the legs, and that of the base. If we are given two side lengths of an isosceles triangle and they are not equal, then the third side must be congruent to one of the two. We are told that this triangle has side lengths 7 and 15, so the third must be 15 or 7. However, by the triangle inequality, the third side cannot be 7; meanwhile, 15 can be confirmed to work. Thus the only possible perimeter is $15 + 15 + 7 = \boxed{37}$.

12. The digit in the seventh position is always removed, and there will always be a digit in the seventh position until there are less than seven digits total, i.e. when there are six; the first six digits are never touched, so the final number will be $\boxed{123456}$.

13. Suppose that $A = 0$. If $X = \times$, then $AXB = 0$ regardless of the value of B, and the value of the expression $AXBYC$ is C, as Y is forced to be $+$. Thus when $A = 0$, $AXBYC = 1$ if $C = 1$ and 2 if $C = 2$. Analogously, when $C = 0$ the value is determined by A. Now suppose that $B = 0$. If $X = \times$, then the value of A doesn't matter, and $AXBYC = C$. If $Y = \times$, then C doesn't matter, so $AXBYC = A$. These can only be 1 or 2, so there are $\boxed{\text{two}}$ possible values.

14. The smaller circle has diameter 2, so the smaller square has side length 2. Thus, the smaller square has diagonal length $2\sqrt{2}$. That diagonal forms the diameter of the larger circle, so its radius is $\sqrt{2}$, and the larger square has side length $2\sqrt{2}$.
The smaller circle has area π. The larger circle has area $\pi\sqrt{2}^2 = 2\pi$, and the smaller square has area $2^2 = 4$, so the total desired area is $(2\pi - 4) + \pi = \boxed{3\pi - 4}$.

15. By observing the first few n^n, we see that the sequence grows quite quickly; $1^1 = 1, 2^2 = 4, 3^3 = 27, 4^4 = 256, 5^5 = 3125$, and so on. In fact, the subsequent one is near our upper limit of 50000: $6^6 = 46656$. $50000 - 46656 = 3344$, so our answer would be 6 if the sum of the previous n^n was less than 3344. However, $4^4 + 5^5 = 256 + 3125 = 3381$, which is too big, so our maximum n must be $\boxed{5}$.

16. Since the radii to the point of contact of tangent circles are tangent, we see that $O_1O_2 = 3, O_1O_3 = 4, O_2O_3 = 5$. This is a right triangle, and has area $\frac{1}{2}3 \times 4 = \boxed{6}$.

17. First, note that $\frac{x+y}{xy} = \frac{1}{x} + \frac{1}{y}$, and the two addends are independent of each other. Thus, we want to minimize them individually. The first is minimized when x is maximized, which is at 2015; similarly, $y = 403$. The final answer is $\frac{1}{2015} + \frac{1}{403}$, or $\boxed{\frac{6}{2015}}$.

18. $6^{12} + 12^6 = 6^6(6^6 + 2^6) = 12^6(3^6 + 1) = 12^6(730) = 10 \times 12^6(73) \implies \boxed{73}$.

19. Suppose the first point is fixed. The second point could be either 60° clockwise or counterclockwise from it, so the total region of the circumference is 120° degrees. $\frac{120^\circ}{360^\circ} = \boxed{\frac{1}{3}}$.

20. This question is solved by carefully plotting each function and noting all intersection points. $y = \frac{1}{x}$ intersects with $y = 1$ at $(1,1)$, with $y = x$ at $(1,1)$ and $(-1,-1)$, and with $y = x^2$ at $(1,1)$. $y = 1$ intersects with $y = x$ at $(1,1)$ and with $y = x^2$ at $(1,1)$ and $(-1,1)$. Finally, $y = x$ intersects with $y = x^2$ at $(1,1)$ and $(0,0)$. Careful counting reveals that there are $\boxed{12}$ pieces.

21. Alex travels the first leg of his trip in $\frac{300}{60} = 5$ hours, the second in $\frac{90}{90} = 1$ hour, and the third in $\frac{360}{40} = 9$ hours. The total length of time is $5 + 1 + 9 = 15$ hours. The total distance is $300 + 90 + 360 = 750$ hours. Thus, his average speed is $\frac{750}{15} = \boxed{50}$ miles per hour.

22. This is the sum of the first 25 triangular numbers. By the Hockey Stick Theorem, this equals $\binom{27}{3} = \boxed{2925}$.

23. The sum of the digits is a multiple of 15, which means that it is also a multiple of 3, so the number itself is divisible by 3. It is also given that this number is a multiple of 35; since 35 is coprime to 3, the number must be a multiple of $3 \times 35 = 105$. Checking the three-digit multiples of 105, we see that $105 \times 7 = \boxed{735}$ satisfies the conditions.

24. We will look at this recursively. Suppose, at some time, the ant is distance $\sqrt{d}$ from the origin. It moves 1 unit perpendicular to its previous position, so the lengths of $\sqrt{d}$ and 1 form the legs of a right triangle; the new distance will be $\sqrt{\sqrt{d}^2 + 1} = \sqrt{d+1}$. After 0 seconds, the distance is $1 = \sqrt{1}$, and so in general, the distance after t seconds will be $\sqrt{t+1}$. After 2015, the distance will be $\sqrt{2016}$, or $\boxed{12\sqrt{14}}$.

25. Suppose the prism's lengths are r, s, t. We are given $rs = 16$, $st = 20$ and $tr = 45$. We want rst. If we multiply the 3 equations given, we get $(rs)(st)(tr) = r^2s^2t^2 = (rst)^2 = (16)(20)(45) \implies rst = \boxed{120}$.

26. Here we will utilize subtractive counting; instead of the probability that the coin will land heads at least twice, we will compute the probability that it will land heads less than two times, and subtract the result from 1. The probability that all flips are tails is $\frac{1}{3}^4 = \frac{1}{81}$, while the probability that there is exactly one heads is $\frac{1}{3}^3 \times \frac{2}{3} \times 4 = \frac{8}{81}$. Thus the probability that there will be less than two heads is $\frac{8}{81} + \frac{1}{81} = \frac{1}{9}$, so our answer is $1 - \frac{1}{9} = \boxed{\frac{8}{9}}$.

27. Let Abhi, Boris and Sandy shovel snow at a rate of a, b and s yards/hour. We are given, then, that $\frac{1}{\frac{1}{a}+\frac{1}{b}} = 3 \implies \frac{1}{a} + \frac{1}{b} = \frac{1}{3}$. Similarly, $\frac{1}{a} + \frac{1}{s} = \frac{1}{4}$ and $\frac{1}{b} + \frac{1}{s} = \frac{1}{5}$. We wish to find $\frac{1}{\frac{1}{a}+\frac{1}{b}+\frac{1}{c}}$. From the three equations derived from the Given, $\frac{1}{a} + \frac{1}{b} + \frac{1}{s} = \frac{1}{2}(\frac{1}{3} + \frac{1}{4} + \frac{1}{5}) = \frac{47}{120}$. So, the answer is $\boxed{\frac{120}{47}}$.

28. We are given that $\frac{a+b+c}{3} = 46$. We want to find $\frac{(a-7)+(b+4)+(c+6)}{3} = \frac{a+b+c+3}{3} = \frac{a+b+c}{3} + 1 = \boxed{47}$. (It turns out that the sequence information was superfluous.)

29. Note that Jon and Alex are themselves contradictory, so one must be a Liar, i.e. there is at least one Liar. Jon is the Liar, and Alex is the Truth-teller. This means that Soonho must also be telling the truth. Now, Mike and Claire are accusing each other, plus Jon (who we already know is one), of being a Liar. We cannot tell which is which, but we can say that exactly one of them is a Liar. Thus there are only $\boxed{\text{two}}$ Liars: Jon and one of Mike and Claire.

30. Suppose $x = -a$ is an x-intercept. Then, $0 = (-a)^2 - 2015|-a| + 20152015 = a^2 - 2015|a| + 20152015$, so $x = a$ is also an x-intercept. The sum of those two values is 0, and since in general the intercepts will come in pairs, the answer is $\boxed{0}$.

31. When it comes to dividing by 8, $5 \equiv -3$, so $5^5 = 5^3 5^2 = (-3)^3 25 \equiv (-3)^3 = -3^3$, so $5^5 + 3^3 \equiv 0$. Similarly, $7^7 \equiv (-1)^7 = -1$, so $1^1 + 7^7 \equiv 0$. $8 \mid 4^4$ and $8 \mid 6^6$, so both of those go to 0. All that is left is $2^2 = \boxed{4}$

32. The area of a regular hexagon with side length s is $6 \times \frac{s^2\sqrt{3}}{4} = \frac{3s^2\sqrt{3}}{2}$. Let a and b be the lengths of the triangle; then $[H_1] + [H_2] = \frac{3\sqrt{3}}{2}(a^2 + b^2)$. By the Pythagorean Theorem, $a^2 + b^2 = c^2 = 36$, so the sum of the areas is $\frac{3\sqrt{3}}{2}(36) = 54\sqrt{3}$. Thus our final answer is $54 + 3 = \boxed{57}$.

33. Write out the sum. Start with $a = 1$: $1 \times 1 + 1 \times 2 + \cdots + 1 \times 9 = 1(1 + 2 + \cdots + 9)$. For $a = 2$, the sum is $2(1 + 2 + \cdots + 9)$. By the distributive property, the final answer is $(1 + 2 + \cdots + 9)(1 + 2 + \cdots + 9) = (45)(45) = \boxed{2025}$.

34. Let AB and O_1O_2 intersect at C, and the distance from A to C be h. As AB and O_1O_2 are perpendicular, this gives us two right triangles (CAO_1 and CAO_2) on which we can apply the Pythagorean Theorem. The area of the quadrilateral that we want is $AB*h$, or $9h$. By applying the Pythagorean Theorem twice, we have the equation $\sqrt{16-h^2}+\sqrt{49-h^2}=9$. Some squaring, rearranging, squaring and cancelling gives us $h^2=\frac{80}{9}$, so $h=\frac{4\sqrt{5}}{3}$, and our area is $\boxed{12\sqrt{5}}$.

35. Time is awkward to graph in two dimensions, but note that the pace is constant, so the distance covered in both 5-minute stretches is the same. Call that distance d.
The set of points reachable in the first 5 minutes is a circle with radius d, centered at BCA. We choose one of those points randomly, and call it P.
The set of points reachable from that P in the next 5 minutes is a circle with radius d, centered at P. We want the points on the boundary of that circle that are outside the first circle, because those points will be further from BCA.
Call Q the point on the first circle diametrically opposite P, and let the two circles intersect at R and S. $PR = PS = d$, $PQ = 2d$. Also, note that $\angle QRP = 90°$, because it is inscribed in a semicircle. So $\triangle QRP$ is a right triangle, and we see that it is a $30-60-90$ triangle. So $\angle RPQ = 60°$, $\angle RPS = 120°$, so minor arc RS is $\frac{1}{3}$ of the circle, and the major arc is thus $\boxed{\frac{2}{3}}$ of the circle.

36. We first have to find the maximum value of $a_0 \times a_1 \times \cdots \times a_k$. For two positive integers A and B, say that A is "more efficient" than B if $A^B > B^A$. We will now show two facts about effciency:

- 3 is the most efficient integer; i.e. $3^N > N^3$ for all $N > 3$.
  Let $f(x) = 3^x$ and $g(x) = x^3$. At $x = 3, f(x) = g(x)$. For all $x > 3$, $\frac{f(x+1)}{f(x)} > \frac{g(x+1)}{g(x)}$, or $3 > \frac{x^3+3x^2+3x+1}{x^3}$, or $2x^3 - 3x^2 - 3x - 1 > 0$, which is true for all $x > 3$.
- 2 is the second-most efficient integer. Similarly to Lemma 1, let $f(x) = 2^x$ and $g(x) = x^2$. At $x = 2, f(x) = g(x)$. For all $x > 3$, $\frac{f(x+1)}{f(x)} > \frac{g(x+1)}{g(x)}$, or $2 > \frac{x^2+2x+1}{x^2}$, or $x^2 - 2x - 1 > 0$, which is true for all $x \geq 3$.

Therefore, we want as many of our a_i's to be 3, and fill up any remaining ones (due to parity) with 2's. This means we want 6703's and 22's, since $(670 \times 3) + (2 \times 2) = 2014$. This gives us $n = \boxed{2}$.

37. We see that $x^2 + 1 = 13x \implies x + \frac{1}{x} = 13$. Let $f(n) = x^n + x^{-n}$ for n an integer. We see that $f(2n) = f(n)^2 - 2$. So, $f(2) = 13^2 - 2 = 167$, and $f(4) = 167^2 - 2$, which ends in $\boxed{7}$.

38. We know that his rolls are some permutation of $a, b, 2a$, with $a \leq b \leq 2a$. We quickly see that $a \leq 3$, so our set of tuples $(a, b, 2a)$ (possibly reordered) is reasonably sized. The possibilities are:

- $(1,1,2)$: 3 arrangements
- $(1,2,2)$: 3 arrangements
- $(2,2,4)$: 3 arrangements
- $(2,3,4)$: 6 arrangements
- $(2,4,4)$: 3 arrangements
- $(3,3,6)$. 3 arrangements
- $(3,4,6)$: 6 arrangements
- $(3,5,6)$: 6 arrangements

- $(3,6,6)$. 3 arrangements

In total, there are 36 rolls that satisfy the condition, out of 216 total possible rolls of the 3 die. Thus, the answer is $\boxed{\frac{1}{6}}$.

39. It takes Rebecca $R(x,y) = \frac{|x|}{20} + \frac{|y|}{10}$ hours to reach point (x,y), and it takes The Great Bustard $GB(x,y) = \frac{\sqrt{x^2+y^2}}{10}$ hours to reach that same point. We wish to consider the set of points (x,y) with $GB(x,y) < R(x,y)$. We will only consider the first quadrant for the time being (as for $f = R$ or $f = GB$, $f(x,y) = f(-x,y) = f(x,-y) = f(-x,-y)$, so we can eliminate the $|\cdot|$ signs).
We want $\frac{x}{20} + \frac{y}{10} > \frac{\sqrt{x^2+y^2}}{10} \implies \frac{x}{2} + y > \sqrt{x^2+y^2} \implies \frac{x^2}{4} + xy + y^2 < x^2 + y^2 \implies xy > \frac{3}{4}x^2 \implies y > \frac{3}{4}x$. We see that this region, in quadrant 1 and restricted to $x, y < 1$, has area $\frac{5}{8}$. A similar figure will occur in the other 4 quadrants, so the desired area is $\boxed{\frac{5}{2}}$.

40. The desired probability is the likelihood that after 9 moves, the ant has moved right 5 steps and left 4. There are $\binom{9}{5} = 126$ ways for these steps to have happened, out of $2^9 = 512$ total paths, so the answer is $\frac{126}{512} = \boxed{\frac{63}{256}}$.

41. Note that the desired sum is equal to $S = (1 + \frac{1}{3} + \frac{1}{9} + \cdots) + (\frac{1}{3} + \frac{4}{9} + \frac{7}{27} + \frac{10}{81} + \cdots) = \frac{1}{1-\frac{1}{3}} + \frac{1}{3} + \frac{1}{3}S \implies \frac{2}{3}S = \frac{3}{2} + \frac{1}{3} = \frac{11}{6} \implies S = \boxed{\frac{11}{4}}$.

42. We will build up the number of ways to reach a given corner. Let the top-left corner be $(0,0)$, and let $w(x,y)$ be the number of ways to reach point (x,y). We see that $w(x,y) = w(x-1,y) + w(x,y-1)$. The only additional pieces of set-up that we need to solve the problem are that $w(x,y) = 0$ whenever $x < 0$ or $y < 0$, $w(0,0) = 1$, and $w(3,3) = 0$ ($(3,3)$ is the interior of the cut-out square). Since there are no irregularities, it turns out that for $x \le 2$ or $y \le 2$, $w(x,y) = \binom{x+y}{x} = \binom{x+y}{y} = w(y,x)$. Now, we have:

$$\begin{gathered} w(3,4) = w(4,3) = 15 \\ w(3,5) = w(5,3) = 36 \\ w(3,6) = w(6,3) = 64 \\ w(4,4) = w(4,4) = 30 \\ w(4,5) = w(5,4) = 66 \\ w(4,6) = w(6,4) = 130 \\ w(5,5) = w(5,5) = 132 \\ w(5,6) = w(6,5) = 262 \\ w(6,6) = w(6,6) = \boxed{524} \end{gathered}$$

43. Using the Law of Cosines, we can find the distance d between the two hands, given the angle θ between them: $d = \sqrt{2^2 + 3^2 - 2(2)(3)\cos\theta}$. Plugging in $d \le \sqrt{7}$, we have $7 \ge 13 - 12\cos\theta \implies -60° < \theta < 60°$, i.e. $\theta \in (0°, 60°) \cup (300°, 360°)$ gives a desired value of d. At 1 o'clock, the hands form a 330° angle; the angle cannot be any larger at any previous point. We know that the minute hand moves continuously away from the hour hand, so every real degree measure from 0° to 330° is met. This means 90° out of 330° possible angles works, which means our final answer is $\frac{90}{330} = \boxed{\frac{3}{11}}$.

44. Multiply through by $ab(a+b)$ to get $0 = b(a+b) - a(a+b) - ab = b^2 - a^2 - ab$. Now, consider the desired sum: $\frac{a^2}{b^2} + \frac{b^2}{a^2} = \frac{a^4+b^4}{a^2b^2}$. Using $0 = b^2 - a^2 - ab$, $b^2 - a^2 = ab \implies b^4 + a^4 - 2a^2b^2 = a^2b^2 \implies b^4 + a^4 = 3a^2b^2$, so plugging in yields $\boxed{3}$.

45. $2015 = 5 \times 13 \times 31$, $2015^{2015} = 5^{2015}13^{2015}31^{2015}$. Any factor of 2015^{2015} is of the form $5^a 13^b 31^c$, and for that factor to be a square, a, b, c, all have to be even. There are 1008 choices for each of a, b, c, so $N = 1008^3 = (2^4 3^2 7)^3 = 2^{12}3^6 7^3$. Perfect square factors of N are of the form $2^a 3^b 7^c$, where a, b, c are all even. There are 7 choices for a, 4 choices for b and 2 choices for c. Thus the desired value is $7 \times 4 \times 2 = \boxed{56}$.

46. Let r and s be the roots of the first equation. Then by Vieta's formulae, $r + s = c$ and $rs = a$. We are given that the roots of the second equation are $r - 1$ and $s - 1$, so again by Vieta's, $(r - 1) + (s - 1) = -a$ and $(r - 1)(s - 1) = b$. Expanding the latter, we have $rs(r + s) + 1 = b$; substituting a for rs and $-c$ for $r + s$ gets us $b = a + c + 1$. This means that the values of $a + b + c$ are the values of $2b - 1$. Meanwhile, substituting a for rs and $-a + 2$ for $r + s$ gives us $b = 2a - 1$, so now we are looking for the values of $4a - 3$.
$rs = a$ and $r + s - 2 = -a$, so $rs + r + s - 2 = 0 \implies rs + r + s + 1 = 3 = (r + 1)(s + 1)$. r and s, and as a result $r + 1$ and $s + 1$, must be integers, and the only two pairs of integers that multiply to 3 are $\{1, 3\}$ and $\{-1, -3\}$. The first pair gives us $r = 0$ and $s = 2$, so $a = rs = 0 \implies 4a - 3 = -3$. The second pair gives us $r = -2$ and $s = -4$, so $a = rs = 8 \implies 4a - 3 = 29$. Thus our two possible values of $a + b + c$ are $\boxed{\text{-3 and -29}}$.

47. We will use the fact that $a + b + c = 1$ to rewrite the equation as

$$\tfrac{1}{1-2a} + \tfrac{1}{1-2b} + \tfrac{1}{1-2c} = 1$$

Multiplying through by $(1 - 2a)(1 - 2b)(1 - 2c)$ yields

$$\begin{gathered}(1 - 2b)(1 - 2c) + (1 - 2a)(1 - 2c) + (1 - 2a)(1 - 2b) = (1 - 2a)(1 - 2b)(1 - 2c)\\ 1 - 2b - 2c + 4bc + 1 - 2a - 2c + 4ab + 1 - 2a - 2b + 4ab = 1 - 2b - 2c + 4bc - 2a + 4ab + 4ac - 8abc\\ 3 - 4a - 4b - 4c = 1 - 2a - 2b - 2c - 8abc\\ -1 = -1 - 8abc\\ \boxed{0} = abc\end{gathered}$$

48. Consider $f(x) = k$ for some real k. Then $\frac{x^2+4x+13}{3x^2+2x+3} = k$ must have solution. If this expression is rewritten as $(3k-13)x^2+(2k-4)x+(3k-1) = 0$, since this has a solution, the discriminant of the quadratic (in x) must be positive: $(k-2)^2-(3k-1)(3k-13) \geq 0 \implies 8k^2-2k+9 \leq 0$. By Vieta's formula, the extremal values of k multiply to $\boxed{\frac{9}{8}}$

49. Let a be the probability that the ball is in the center, b be the probability that it is in a cube adjacent to the center, c be the probability that it is adjacent to the cubes with likelihood b (but is not in the center), and d be the probability that is is in a corner cube. We see that $a + b + c + d = 1$; $a = \frac{1}{5}b$ since for any 'b' cube, there are 5 adjacent cubes, but only one is the center; $b = a + \frac{1}{2}c$, since for any 'c' cube, there are 4 adjacent cubes, 2 of which are 'b' cubes; and $d = \frac{1}{2}c$. Solving the system yields $b = 5a$, $c = 8a$, $d = 4a$. Therefore, $a = \frac{1}{1+5+8+4} = \boxed{\frac{1}{18}}$

50. Let the dimension of the box be $a \times b \times c$. For every plane that does not contain the origin, there is a line intersecting it, and there are $a + b + c$ planes giving us $a + b + c$ cubes intersected, Now, every $gcd(a, b)$ or $gcd(a, c)$ or $gcd(b, c)$ the diagonal intersects a edge of one of the unit cubes, reducing number of boxes intersected by 1. But there are overlaps, $gcd(a, b, c)$ intersects a corner of the unit cube, thereby reducing the number of boxes intersected by 2. Since pairwise gcd over-counts, we add $gcd(a, b, c)$. Our desired foruma becomes $a + b + c - gcd(a, b) - gcd(a, c) - gcd(b, c) + gcd(a, b, c)$, and plugging in the values given yields $\boxed{306}$.