

Joe Holbrook Memorial Math Competition

6th Grade

October 9th, 2016

General Rules

- You will have **90 minutes** to solve **50** questions. Your score is the number of correct answers.
- Only answers recorded on the answer sheet will be graded.
- This is an individual test. Anyone caught communicating with another student will be removed from the exam.
- Scores will be posted on the website. Please do not forget your ID number, as that will be the sole means of identification for the scores.
- You may **not** use the following aids:
 - Calculator or other computing device
 - Compass
 - Protractor
 - Ruler or straightedge

In addition, you must use the scrap paper supplied by the proctors.

Other Notes

- Write legibly. If the graders cannot read your answer, you will be given no credit for that question.
- Fractions should be written in **lowest terms**. Please convert all mixed numbers into **improper fractions**.
- For constants such as e or π , do not approximate your answer: for example, if the answer to a question is 7π , then you should **not** write 22 or 21.99.
- You do not need to write units in your answers.
- Rationalize all denominators. In addition, numbers within a square root must be squarefree, e.g. $\sqrt{63}$ should be written as $3\sqrt{7}$.
- Ties will be broken by the number of correct responses to questions 41 through 50. Further ties will be broken by the number of correct responses in the last five questions.

1. What is $-17 \cdot -16 \cdot -15 \cdots 15 \cdot 16 \cdot 17$?
2. What is the probability of getting an even number when a standard six-sided die is rolled?
3. If Yousun is 5 feet tall and Youjung is 6 inches taller than Yousun, how tall is Youjung in inches?
4. A phone has a maximum battery life of 10 hours. There is 12% battery left. How many minutes are left until the phone dies?
5. Find the greatest common factor of 2016 and 2772.
6. What is $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{6}$?
7. What is the smallest integer n such that $2^n > 2016$?
8. One of Jake's cake recipes calls for 1.5 cups of milk, 2 eggs, and 1 cup of butter. However, since he is having a party tonight, he has decided to triple his recipe. What is the total volume of milk and butter used?
9. Harry Hounini, the famous magician, invents a magic trick, and he hopes you will test it out for him. This is how the trick works: first think of any number. Now add that number to 2016, and multiply the sum by 4. Now subtract 12 from the product, and divide the result by 4. Lastly, subtract off the original number. Incredibly, Hounini knows exactly what that number is. What is it?
10. The number of lily pads in Kelvin the Frog's pond doubles every day. If there were 48 lily pads on Saturday, on what day of the week did he have an odd number of lily pads?
11. A fruit smoothie has 10% water. A soda has 2% water. If 1 liter of the smoothie and 1 liter of the soda are mixed, what percent of the resulting solution is water?
12. Arthur the Aardvark is an aspiring marathon runner. He wants to run a half-marathon, 13 miles, at an average speed of 10 miles per hour. She has run the first 6 miles in 50 minutes. How many miles per hour should she run at for the remainder of the race to meet her goal?
13. January 1st, 2016 was a Friday. What day of the week is the 2016th day after January 1st, 2016?
14. What is the largest integer that, when cubed, is less than 1,000,000?
15. David and June went to a pizza restaurant together. David ordered 3 slices of plain pizza and 2 slices of pepperoni pizza. June ordered 2 slices of plain pizza and 4 slices of pepperoni pizza. If David and June paid \$25 in total and a slice of pepperoni pizza costs 50 cents more than a slice of plain pizza, how much did David pay?
16. If $f(x) = x + 1$, $g(x) = \lfloor x \rfloor$, and $h(x) = g(f(x))$, compute $h(\pi^2)$.
17. Jake goes to the grocery store because he needs to buy milk, eggs, and butter for a cake recipe. If there are 3 different brands of milk, 2 different brands of eggs, and 4 different brands of butter, and Jake only wants to buy one of each item, how many different combinations of milk eggs and butter can he buy?
18. Suppose 3 flips are worth 5 flops and 9 flops are worth 14 flaps. How many flaps are equal to 54 flips?
19. Andrew, Ben, Caleb, Dennis, and Eddie are working to build a house. Sadly, Andrew and Dennis are lazy, so they only work for 1 hour. They are also mean, so the other three people don't work with them. While Andrew and Dennis work alone, they finish only 3% of the house before quitting. Caleb, Dennis, and Eddie working together work 5 times as fast. How many minutes will it take them to finish building the house?
20. Arthur and Sunny are running a 100 meter race against each other. For the first 5 seconds, Arthur runs at 8 m/s. Tired, he slows down to 3 m/s for the rest of the race. Meanwhile, Sunny opened the first 8 seconds of the race running only at 4 m/s. If he wants to at least tie Arthur, what is the minimum speed he must run at for the rest of the race?.
21. Compute the units digit of $2^{2016} + 3^{2016}$.
22. The area of a square is 25. The area of an equilateral triangle is $9\sqrt{3}$. What is the difference in side lengths?
23. Let A , B , and C be the number of positive factors of 2015, 2016, and 2017, respectively. Find the average of A , B , and C .
24. What is the least positive integer value of n such that $|n! - n^3| > 2016$?

25. A square on the Cartesian plane has adjacent vertices on the origin and at $(1, 0)$. It is spun 180° about the origin. What is the total area that is covered by the square during the turn?
26. What is the sum of the largest five digit number and smallest five digit number Tom can form, given that adjacent digits may not be both odd or both even, and adjacent digits must be at least two apart?
27. Arthur the Aardvark dreams of being a basketball player, so he is working on his free throws. On his first 48 attempts, he has only made one basket. How many consecutive baskets must he make so that he improves his shooting percent to his goal of 6%?
28. Two similar right triangles have areas of 40cm^2 and 360cm^2 respectively. If the smaller has a hypotenuse of length 15cm, what is the length, in centimeters, of the hypotenuse of the larger triangle?
29. There are 80 houses in a town called Katzville. The houses are numbered from 1 to 80. The houses can be occupied by humans, Kats, both humans and Kats, or neither. The humans live in houses divisible by 7. The Kats live in houses divisible by 3. Find the sum of the numbers of the houses in Katzville that have only 1 human or 1 Kat living in it.
30. Let $*x*$ be a function defined for positive reals such that $*x* = x$ if $x \geq 1$ and $*x* = \frac{1}{x}$ if $0 < x \leq 1$. If $\frac{*x*}{2} = *x + \frac{1}{2}$, find the sum of all possible x .
31. How many distinct derangements are there for the word *JHMMC*? (A derangement is a permutation such that no element returns to their original position.)
32. A 150 liter water tank is empty. One pipe fills the tank, and a hole has been cut through the bottom of the tank. The pipe fills the tank at a rate of 4 liters per minute, and water drips from the hole at a rate of 2.5 liters per minute. How many seconds will it take to fill the entire tank?
33. We are given a list of seven numbers a, b, c, d, e, f, g , such that the average of the first four numbers is 7 and the average of the last four numbers is 4. Given that the average of all 7 numbers in the set is 5, what is the value of d ?
34. It takes 10 workers 10 days to build a wall. In their latest project, they are supposed to build 5 walls. After the first 2 days, the boss realizes they will not finish by the deadline. How many workers must he add so that he can finish before the 10-day deadline?
35. A fight has broken out in the classroom and Dr. Abramson must rearrange his students' desks to keep the trouble-makers apart! Dr. Abramson's classroom has five students sitting in a single row. However, Ben will only agree to sit next to his friends Caleb and Eddie. Moreover, Andrew and Dennis are feuding and refuse to sit next to each other. In how many ways can Dr. Abramson order his students in a seating arrangement that keeps everybody happy?
36. Compute $|x + y|$ given that $(x + y)^4 - 4 \cdot (x + y)^2 = 1$ and $(x + y)^4 + 4 \cdot (x + y)^2 = 3$.
37. Everyone in DrizzleLand has a 1 in 150 chance of having the muggy virus. The test for this virus has a 96% of giving the correct result for any patient, whether or not he or she has the virus. If Zach tests positive, what is the probability that he actually has the muggy virus?
38. David Ni went to buy shirts from the store. Some of the shirts were \$10, some were \$30, and some were even \$40. If he bought a total of 12 shirts, at least one of each kind, and spent a total of \$240, how many of the expensive \$40 shirts did he buy?
39. Arthur the Aardvark is chained to the top-left corner of a regular hexagonal barn with side length 15 feet. The chain can extend 20 feet in any direction. Find the area that she can roam.
40. Two trains, located 1 kilometer apart, head towards each other. A bird starts flying from the top of one train towards the other. Each time the bird encounters a train, it turns around and flies back. Given that the two trains travel at 10 meters per second, and the bird flies at 15 meters per second, find the total distance the bird travels in meters.
41. Arthur the Aardvark is eating dumplings. He has 3 identical red dumplings, 2 identical yellow dumplings, and 800 identical green dumplings. How many different ways can he choose 6 dumplings to eat for dinner?

42. There are 4 different and unique cats in my house. I have 9 cat toys which are all the same shapes and sizes. Each cat is required to have at least one cat toy to entertain themselves with. How many ways can I distribute the cat toys?
43. What is the largest prime factor of $56^3 - 7^3 - 24^3 - 25^3$?
44. Compute the sum $\frac{7}{\sqrt{1} + \sqrt{2}} + \frac{7}{\sqrt{2} + \sqrt{3}} + \frac{7}{\sqrt{3} + \sqrt{4}} + \cdots + \frac{7}{\sqrt{48} + \sqrt{49}}$.
45. Let w, x, y , and z be positive integers such that $w^2 + x^2 + y^2 + z^2 = 324$. What is the greatest possible value of $w + x + y + z$?
46. What is the total length of the shortest path that goes from $(0, 4)$ to a point on the x-axis, then to a point on the line $y = 6$, then to the point $(18, 4)$?
47. Hannah is sitting at a round table with eight seats. She will be happy if she sits next to at least one of Anna or Savannah, and Anna will be happy if she does not sit next to Savannah. If the three girls and five others fill up all the seats at the table, for how many arrangements will both Hannah and Anna be happy?
48. Consider the base 10 number number $N = 2016(2016(2016(\dots(2016(2016 + 1))\dots)))$, which contains 2015 2016's. How many 1's are in the base 2016 representation of N ?
49. Find the minimum value of $xy + xz + yz$ given that the sum of the squares of the real numbers x, y , and z is 9.
50. Alex the Kat is playing with his magical coloured yarn balls. They come in three colours: red, yellow, and blue. Every hour, the balls multiply and change colour with the following probabilities:
- A red ball has probability $\frac{5}{12}$ of becoming two red balls, $\frac{1}{3}$ of becoming one red ball and one yellow ball, and $\frac{1}{4}$ of becoming one blue ball.
 - A yellow ball has probability $\frac{1}{2}$ of becoming two yellow balls, $\frac{1}{4}$ of becoming one yellow ball, and $\frac{1}{4}$ of becoming one blue ball;
 - A blue ball does not change colour.

If Alex starts with one red yarn ball, what is the probability that eventually all balls will be blue?