5th Grade Solutions

1. We can simply count by 17's. The first couple multiples of 17 are 17, 34, 51, 68, and 85. Therefore, there are **5** multiples of 17 less than 100. OR We note that $100 \div 17 = 5.88\ldots$, or $100 = 17 \times 5.88\ldots$. Therefore, 17×1; 17×2; 17×3; 17×4; 17×5 are all the multiples of 17 smaller than 100. This means the answer is **5**.

2. $25 + 16 + 9 + 4 + 1 = \mathbf{55}$

3. Sungjae and David have $64 - 28 = 36$ marbles together, so David has $38 \div 2 = \mathbf{18}$ marbles.

4. Because Eric is 19, Dennis is $19 + 6 = 25$, so I am $25 - 4 = \mathbf{21}$ years old.

5. The xth term is $x^2 + 1$. Therefore the next number is for $x = 8$, **65**.

6. Since he places 20 books in the fridge, he must remove $20 \times 9 = 180$ pieces of candy in total. Subtracting 180 from the total number of candies gives us **220** pieces.

7. One lap is $\frac{1}{4}$ of a mile, so 4 laps is one mile. Since the runner needs to run 26 miles, he must run $26 \times 4 = \mathbf{104}$ laps.

8. Our answer is $6446 \div 11 = \mathbf{586}$.

9. There are $6 \times 6 = 36$ ways to roll two dice, and there are two ways to roll a sum of 11 (5 and 6 or 6 and 5), a probability of $\frac{2}{36}$ or $\mathbf{\frac{1}{18}}$.

10. The number of ways to obtain 2 is 1 $(1,1)$. The number of ways to obtain 3 is 2 (1;2; 2,1). Continuing in this fashion until 7, we find that the number of ways to obtain 7 is 6. Past 7, the number of ways decreases again (there are only 5 ways to get a sum of 8, 4 ways to get a sum of 9, etc.) Thus, **7** has the most number of ways to be obtained and is therefore the most likely sum.

11. Using the formula, our answer is $\frac{9}{5}C + 32 = \frac{9}{5}(45) + 32 = 9 \times 9 + 32 = 81 + 32 = \mathbf{113}°F$

12. We first note that $\frac{64}{26}$ is greater than 2 but less than 3. Since it will take 78 Gigantic men to slay 3 dragons, they do not have enough men to slay a third dragon, so the answer is **2**.

13. From trial and error, one can easily deduce that the three numbers are 9, 10, and 11. Therefore, the answer is $9 \times 10 \times 11 = \mathbf{990}$.

14. Each slice of plain pie cost \$1. Steven paid \$1 for the whipped cream in addition to \$5 for the five slices of pie that he ate, for a total of \$6. James paid only \$3 for the three slices of pie that he ate. Hence Steven paid $6 - 3 = \mathbf{3}$ dollars more than James.

15. 40% is equal to $\frac{2}{5}$. $\frac{2}{5} \times \frac{13}{4} = \mathbf{\frac{13}{10}}$ OR **1.3**

16. We can work backwards to find the answer. Since she added, we can reverse that by subtracting: the original number was $26 - 9 = 17$. She was supposed to multiply this by 9, so the answer is $17 \times 9 = \mathbf{153}$.

17. There are $6 \times 8 = 48$ slices in 6 pies, so Jason would have paid $48 \times 1.50 = 72$ dollars if he bought individual slices. However, he actually paid $10 \times 6 = 60$ dollars, so he saved $72 - 60 = \mathbf{12}$ dollars.

18. There are 25 people in front of Nate and 25 people behind him. Including him, $25 + 25 + 1 = \mathbf{51}$.

19. The circumference of a circle with radius of length r is $2\pi r$. So, our answer is $2\pi \times 16 = 32\pi \approx 32 \times 3.14159 = 100.53$, which rounds up to **101**.

20. Each of the four players has an equal chance of getting the ace of diamonds, so you have a $\frac{13}{52}$ chance of getting the ace of diamonds. Simplify $\frac{13}{52}$ by dividing the top and bottom by 13 and you have a $\mathbf{\frac{1}{4}}$ chance of getting the ace of diamonds.

21. Since there are 4 packages of hot dogs, there must be $4 \times 6 = 24$ hot dogs in total. Since there are 24 hot dogs in total, 24 buns are needed. Since there are 8 buns to a package, **3** packages are needed $(3 \times 8 = 24)$.

22. Follow the pattern: the fourth day he will eat 40 Veggie loops and so on until the last day, the 7th day, he will eat 70. So $10+20+30+40+50+60+70 = \frac{10+70}{2} \times 7 = \frac{80}{2} \times 7 = 40 \times 7 = \mathbf{280}$.

23. First, find the common denominator. From this, we get $\frac{3\times7}{3\times5} - \frac{5\times1}{5\times3} = \frac{21-5}{15} = \mathbf{\frac{16}{15}}$.

24. Note that $\frac{1}{3} = \frac{1}{1\times3}$, $\frac{1}{8} = \frac{1}{2\times4}$, $\ldots$, $\frac{1}{80} = \frac{1}{8\times10}$. The nth term in this sequence can be represented as $\frac{1}{n(n+2)} = \frac{1}{2}\left(\frac{1}{n} - \frac{1}{n+2}\right)$. Hence, the given sum is equal to half of
$\frac{1}{1} - \frac{1}{3} + \frac{1}{2} - \frac{1}{4} + \frac{1}{3} - \frac{1}{5} + \frac{1}{4} - \frac{1}{6} + \frac{1}{5} - \frac{1}{7} + \frac{1}{6} - \frac{1}{8} + \frac{1}{7} - \frac{1}{9} + \frac{1}{8} - \frac{1}{10} = \frac{1}{1} + \frac{1}{2} - \frac{1}{9} - \frac{1}{10} = \frac{58}{45}$, which is $\mathbf{\frac{29}{45}}$.

25. $25 \times 4 \times 25 \times 4 \times 64 = 100 \times 100 \times 64 = \mathbf{640,000}$

26. Note that $30 = 1 \times 30$, $60 = 2 \times 30$, $90 = 3 \times 30$ and $120 = 4 \times 30$. So, $30 + 60 + 90 + 120 = 30(1 + 2 + 3 + 4)$. Therefore, the answer is **30**.

27. If both numbers are multiples of the same number, then their difference should also be a multiple of the same number (this can be verified with the distributive property of numbers). $551 - 437 = 114$. 114 factors into 2, 3, and 19. 551 and 437 are not multiples of 3 or 2, so they must be multiples of 19. This can be verified by dividing both numbers by **19**.

28. The surface area of a cube of side length s is $6s^2$ since a cube has six square faces. When $6s^2 = 384 = 6 \times 64 = 6 \times 8^2$, the side of the cube must be equal to 8. The volume of a cube with side length 8 is $8^3 = \mathbf{512}$.

29. By getting a 20% discount off of \$40, we will save 20% of \$40, which is $40 \times \frac{20}{100} = \mathbf{8}$ dollars.

30. She collects $6 - 4 = 2$ real coins each time, so she must go **13** times to get 26 coins because 12 times (24 coins) is not enough.

31. If Isabel eats $\frac{1}{5}$ of the pizza, there is $\frac{4}{5}$ of the pizza left. If Jie ate half of what was left, he ate $\frac{2}{5}$ of the pizza. Since they only ate fifths of the pizza, the smallest number of slices is **5**.

32. $\dfrac{10 \times 24 \times 18 \times 8}{20 \times 9 \times 12 \times 16} = \dfrac{10 \times 2 \times 12 \times 9 \times 2 \times 8}{10 \times 2 \times 12 \times 9 \times 2 \times 8} = \mathbf{1}$

33. Let x be the largest integer, meaning that the least integer is then $x - 24$. If we sum up these 25 integers, we get $25x - (1+2+3+\cdots+23+24) = 25x - \frac{(24)(25)}{2} = 25x - 300$. This is equal to 700, so $25x = 1000$ meaning $x = \mathbf{40}$.

34. The sum of all the numbers in the list is $6 \times 64 = 384$. The sum of the first two numbers is $2 \times 26 = 52$, so the sum of the last four numbers is $384 - 52 = 332$. The average of these numbers is $332 \div 4 = \mathbf{83}$.

35. Start with the two most obvious hints, which are that Bob is directly after Eve, and Caroll is directly behind Dave. We are given two blocks then, with one person left. Since Alice has to be next to Dave, and since the spot directly behind Dave is taken, she must be in front of Dave. Thus we have two blocks now from front to back: Alice, Dave, Caroll and Eve, Bob. Since one of the clues indicated that Eve was in front of Caroll, we have that Eve, Bob is in front of Alice, Dave Caroll and thus we have Eve, Bob, Alice, Dave, Caroll. The middle person is **Alice**.

36. If $10^i < n < 10^{(i+1)}$ then the sum of n's digits is at most $9i$, which is less than 10^i for $i > 1$. So $n < 10$ and then n must equal **7**.

37. We can look for factors of 30 in pairs: 1,30; 2,15; 3,10; 5,6. Thus, we have **8** numbers that divide 30 evenly.

38. We take the first equation and subtract it from the second: $(x+2y)-(x+y) = 7-3$ Thus, $y = 4$. Substituting into the first equation, $x + 4 = 3$ gives $x = -1$. Now substituting into the final expression, we find that $-1 + 3(4) = \mathbf{11}$

39. The formula for the area of an equilateral triangle is $s^2\frac{\sqrt{3}}{4}$ where the side length is equal to s. To find the area, simply substitute the s with 4, giving $4\sqrt{3}$. OR Divide the equilateral triangle and in half so that two right triangles are made. Then, using the Pythagorean theorem and the knowledge of what the hypotenuse (the side length) and what one of the legs (half of the side length) are, the height of the triangle can be found. This knowledge can then be applied to the normal area of a triangle formula $\frac{1}{2}bh$, in which the base is one of the side lengths, giving $\frac{1}{2}4 \times 2\sqrt{3} = \mathbf{4\sqrt{3}}$.

40. The number of cupcakes consumed by Jongwhan is equal to $10 + 14 + \cdots + (10 + 4 \times 29) = 10 \times 30 + 4 \times (1 + 2 + ... + 29) = 10 \times 30 + 4 \times (30 \times 14 + 15) = 2040$, and the number of cupcakes consumed by Michael is equal to $100 + 97 + \ + (100 - 3 \times 29) = 10 \times 30 - 3 \times (30 \times 14.5) = 1695$. Hence, the sum is $2040 + 1695 = \mathbf{3735}$.

41. Since we need to eliminate 99 members of the math team to find the winner, **99** games are required.

42. Writing out the prime factorization of the numbers $14, 16, 8$, we have $14 = 2 \cdot 7, 16 = 2^4, 8 = 2^3$. From this, it is easy to deduce that $gcd(14, 16, 8) = 2$ and $lcm(14, 16, 8) = 16 \times 7 = 112$. This yields $gcd(14, 16, 8) \times lcm(14, 16, 8) = 224$.

43. It may seem that the snail goes 1 meter everyday, and that he reaches 6 meters after 6 days. However, the snail only slips at night if he is still in the hole. The snail reaches 6 meters from the 4 meter height he began the **5**th day on.

44. Write out the powers of two starting with 2^1. Notice that the last digits form the sequence $2; 4; 8; 6; 2; 4; 8; 6 \ldots$ and that every fourth power of two has 6 as its last digit. Therefore, since 64 is a multiple of 4, **6** is the last digit of 2^{64}.

45. Guess-and-check can be used to solve this problem. OR Let c be the number of questions Quinn got right. Then he got $(25 - c)$ questions wrong. So his score is $10c - 2.5(25 - c) = 150$. Then $10c - 62.5 + 2.5c = 150$ and $12.5c = 212.5$. Therefore, $c = \mathbf{17}$.

46. There are originally 120 cupcakes. The number of red velvet cupcakes is $120 \times \frac{3}{5} = 72$. The number of red velvet cupcakes with red icing is $72 \times \frac{1}{3} = 24$. The number of red velvet cupcakes with red icing that have red sprinkles is $24 \times \frac{1}{2} = 12$. Now, as Ryan gives away 1 of the red velvet cupcakes with red icing and red sprinkles to Jason, the number of leftover red velvet cupcakes with red icing and red sprinkles is $12 - 1 = \mathbf{11}$.

47. Let x equal the number of cows on Farmer Justin's farm and y equal the number of chickens. Set up 2 equations: $x + y = 45$, $4x + 2y = 126$. Multiply the first equation by 2 so it becomes $2x + 2y = 90$. Subtract the first equation from the second equation so the resulting equation is $2x = 36$. Divide both sides by 2 to find what the value of x, or the number of cows on Justin's farm. $x = \mathbf{18}$.

48. Eric's first draw will be something he wants, since he needs all of them, so the probability for the first is $\frac{5}{5}$. For his second draw, he needs 4 kinds of cards, but can draw 5, so his chances for the second are $\frac{4}{5}$. This pattern continues so that the chances of him successfully drawing one card of each type is $\frac{5}{5} \times \frac{4}{5} \times \frac{3}{5} \times \frac{2}{5} \times \frac{1}{5} = \mathbf{\frac{24}{625}}$.

49. Take the 4 different cases as 4 different problems for the time being. If Kevin can carry one bag of chips, one candy bar, and one soda, he must choose one of each. There are 5 choices for chips, 6 choices for a candy bar, and 4 choices for soda, and thus 120 choices for this case. If he can only carry 4 candy bars, then the problem becomes how many ways to choose 4 from a selection of 6, which is ${}_6C_4$ or $\frac{6!}{4!(6-2)!} = 15$. By that same logic, carrying 3 bags of chips is ${}_5C_3 = 10$, and 2 cans of soda is ${}_4C_2 = 6$. The total is $120 + 15 + 10 + 6 = \mathbf{151}$.

50. The only way to have a product of two numbers be odd is if both factors are odd. Thus a and c are both odd, which yields $a = 3$ and $c = 5$. This gives $b = \mathbf{4}$.