1 Grade 4 Solutions

1. $2 + 5 \cdot 6 = 2 + 30 = \boxed{32}$.

2. At the end, Sue is on rung number $9+6-2+3-9+11 = 18$. Since this is also the top rung, there must be $\boxed{18}$ rungs in total.

3. The answer is $\boxed{2}$.

4. $3 \cdot (48 - 2) + 3^2 \cdot 5 - 2 = 3 \cdot 46 + 45 - 2 = \boxed{181}$.

5. $\dfrac{0.4 + 0.04 + 0.004 + 0.0004}{4} = \dfrac{0.4444}{4} = \boxed{0.1111}$.

6. If 5.5 cups are needed for 2 loaves of bread, then $\dfrac{5.5}{2} = 2.75$ cups are needed for 1 loaf of bread. Thus, $2.75 \cdot 5 = 13.75 = \boxed{\dfrac{55}{4}}$ cups are needed for 5 loaves of bread.

7. In the worst case scenario, Chef J grabs 2 chips and 2 truffles. However, whichever chocolate he picks next, there are going to be at least 3 pieces of the same type of chocolate. Therefore, the answer is $\boxed{5}$.

8. $1.354 + 0.79 + 2.005 + 1.8 + 4.05 + 0.001 = \boxed{10}$.

9. There are $\boxed{9}$ such numbers. We can find them simply by listing them all out: 2011, 2101, 2110, 1201, 1210, 1120, 1102, 1012, and 1021.

10. It is easy to see that the largest prime less than 93 is 89. Therefore, the answer is $93-89 = \boxed{4}$ dollars.

11. $\dfrac{1}{2} + \dfrac{1}{4} + \dfrac{1}{8} + \dfrac{1}{16} + \dfrac{1}{32} = \dfrac{16+8+4+2+1}{32} = \boxed{\dfrac{31}{32}}$.

12. The final volume of the potion is the sum of the volumes of the ingredients, so we wish to find $2\frac{1}{2} + \frac{1}{8} + \frac{11}{4} + \frac{2}{3}$. To find this sum, we first write $2\frac{1}{2}$ as a mixed fraction: $\frac{5}{2}$. We then put it all over a common denominator, as follows: $\dfrac{5}{2} + \dfrac{1}{8} + \dfrac{11}{4} + \dfrac{2}{3} = \dfrac{60}{24} + \dfrac{3}{24} + \dfrac{66}{24} + \dfrac{16}{24} = \boxed{\dfrac{145}{24}}$.

13. The total volume of the tank is $1 \cdot 2 \cdot 4 = 8$ cubic feet. Since Jessica's body volume is 1 cubic foot, the answer is $\dfrac{1}{8} \cdot 100 = \boxed{12.5}$ percent.

14. $11^4 = 11^2 \cdot 11 \cdot 11 = 121 \cdot 11 \cdot 11 = 1331 \cdot 11 = \boxed{14641}$.

15. Notice that $22x + 54 = 2(11x + 27)$. The answer is thus $1234321 \cdot 2 = \boxed{2468642}$.

16. Let the palindrome be $abccba$. There are 9 choices for a (1-9), and 10 choices each for b and c (0-9). This gives a total of $9 \cdot 10 \cdot 10 = \boxed{900}$ 6 -digit palindromes.

17. Since x^2y is constant for all values of x and y, we have that x^2y is always equal to $6^2 \cdot 4 = 144$. Therefore, we need y such that $4^2 \cdot y = 144$, so $y = \boxed{9}$.

18. Since $\left|x + \dfrac{5}{2}\right|$ is never negative, there are $\boxed{0}$ solutions.

19. Split the 6 rabbits into two groups of three rabbits. In three minutes, each of the two groups of rabbits willl have eaten 3 carrots, so after $\boxed{\text{3 minutes}}$, they will have eaten 6 carrots in total.

20. If 60% of the people drop out, then 40% of them will take the test. Thus, if x is the number of people they must enlist, we have $0.4x = 1000 \implies x = \boxed{2500}$.

21. Notice that the sum of the integers from $-n$ to n is exactly 0. We can verify this by grouping each number with its opposite and noticing that each pair adds up to 0. Thus, the sum of the numbers from -15 to 15 is 0. If we include the number 16 into our sum, we find that the sum of the numbers from -15 to 15 is 16. There are 32 consecutive integers between -15 and 16 inclusive, so the answer is $\boxed{32}$.

22. After every 20 minutes, the number of parasprites doubles: each of the original parasprites remains, and each of them spawns a new parasprite. Since six sets of 20 minutes pass in two hours, there will be a total of $2^6 = \boxed{64}$ parasprites after 2 hours.

23. There are 20 jelly beans in total, with 4 of them blue. The probability of the first not being blue is $\frac{16}{20}$, the probability of the second not being blue is $\frac{15}{19}$, and the probability of the third not being blue is $\frac{14}{18}$. Thus the probability of getting three non-blue jelly beans is $\frac{16}{20} \cdot \frac{15}{19} \cdot \frac{14}{18} = \boxed{\frac{28}{57}}$.

24. If Granny knits two sweaters, she will have to make $2 \cdot 2100 = 4200$ stitches in total. It takes $\frac{4200}{20} = 210$ minutes to make these stitches, which amounts to $\boxed{\text{3.5 hours.}}$

25. There are 4 choices for the first person, 3 for the second, 2 for the third, and 1 for the fourth. Thus, there are $4 \cdot 3 \cdot 2 \cdot 1 = \boxed{24}$ ways to line up the people.

26. It can be easily verified that the statement holds true when $n = 3, 4,$ or 5. For $n > 5$, it can easily be seen that the longest diagonal is longer than the shortest. Thus, our answers are $\boxed{3, 4, \text{ and } 5}$.

27. If n leaves a remainder of 1 when divided by 2,3,4,5, and 6, $n - 1$ must leave a remainder of 0 when divided by 2,3,4,5,6, that is, 2,3,4,5, and 6 must all divide $n - 1$. This is the same thing as saying that $\text{lcm}(2, 3, 4, 5, 6) = 60$ divides $n - 1$. The smallest such n larger than 1 is therefore $\boxed{61}$.

28. Rewrite this with each of the bases as 2. $8^{3x+4} = 2^{3(3x+4)} = 2^{9x+12}$ and $16^{5x} = 2^{4(5x)} = 2^{20x}$. Equating exponents, we have $20x = 9x + 12 \implies \boxed{x = \frac{12}{11}}$.

29. We know that all squares of real numbers are nonnegative. Thus, if two squares add to 0, both of them must be zero. We must therefore have $2x+3y = 5$ and $x-2y = -7$. Tripling the second equation and adding it to twice the first equation gives us $7x = -11 \implies x = \boxed{-\frac{11}{7}}$.

30. If the product of three numbers is odd, each individual number must be odd. The probability that any single roll of a die results in an odd number is $\frac{1}{2}$, so our answer is $\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \boxed{\frac{1}{8}}$.

31. We have $7a9 + 2b4 = cd13$. Since there is a carry in the ones digit, we must have either $a+b+1 = 1$ or $a+b+1 = 11$. In the former case, $7a9+2b4 < 1000$, which is impossible since $cd13$ is a four-digit number. Thus, $a + b = 11$, whence c and d must be 1 and 0, respectively, so our answer is $10 + 1 + 0 = \boxed{11}$.

32. Rewrite $33.333\ldots\%$ as $\frac{1}{3}$. If $\frac{1}{3}$ of y is 20, then $y = 60$. The sum of the factors of y, namely $1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60$, is $\boxed{168}$.

33. There are no such palindromes. Suppose, for the sake of contradiction there exists a palindrome with 8 digits and whose digits sum to 15 be $abcddcba$. Then we have that $2(a + b + c + d) = 15$, which is impossible since 15 is even, so our answer is $\boxed{0}$.

34. The resulting figure has one circular face, of surface area πr^2, one lateral surface of a cylinder, with surface area $2\pi rh$, and one-half of a sphere, which has surface area $\frac{1}{2}(4\pi r^2) = 2\pi r^2$. Thus, the total surface area is $3\pi r^2 + 2\pi rh = 3\pi(5^2) + 2\pi(5)(10) = \boxed{175\pi}$.

35. There are $\binom{11}{1}$ ways to place the geometry book. Having placed it, there are $\binom{10}{6}$ ways to place the algebra books. Having placed them, there are $\binom{4}{4}$ ways to place the combinatorics books, where $\binom{a}{b} = \frac{a!}{b!(a-b)!}$. Thus there are $\frac{11!}{10!1!} \cdot \frac{10!}{6!4!} \cdot \frac{4!}{4!0!} = \boxed{2310}$ ways to arrange the books on the bookshelf.

36. There are 3 distinct combinations for the first and last letters. The word can be of the form O _ _ _ _ E, O _ _ _ _ O , or E _ _ _ _ O. In all of the cases the four remaining letters are distinct and can be arranged in any order. There are 4 choices for the second letter, 3 choices for the third letter, 2 choices for the fourth letter, and 1 choice for the fifth letter. Thus, there are $3 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = \boxed{72}$ distinct ways to arrange the letters.

37. First of all, we must consider that 2012 is a leap year so we must consider the extra day (hence 2012 will have 366 days instead of 365). We must also consider that October 16, 2011, the day of the contest, is a Sunday. One year from now, which is 366 days later, October 16, 2012 will be in 52 weeks and 2 days, so October 16 will be a Tuesday. Therefore, October 17 will be a $\boxed{\text{Wednesday}}$.

38. If n^2 is a cube, n must also be a cube; if n^3 is a square, then n must also be a square. Thus, n must be both a square and a cube, so it must be a sixth power. The smallest sixth power larger than 1 is $2^6 = \boxed{64}$.

39. After a cycle of one day and one night, the snail has a net movement of 2 meters up the tube. After the ninth day, the snail is $8 \cdot 2 + 3 = 19$ meters from the bottom of the tube. During the ninth night, the snail sinks 1 meter down, and reaches the top of the 20 meter tube on the tenth day. Therefore, the answer is $\boxed{10}$.

40. It is easy to see that the number n appears between positions $(1 + 2 + \cdots + (n-1)) + 1$ and $(1+2+\cdots+n)$ inclusive, that is, $\frac{n(n-1)}{2} + 1$ and $\frac{n(n+1)}{2}$ inclusive. Since 200 lies between $190 = \frac{20 \cdot 19}{2} + 1$ and $210 = \frac{20 \cdot 21}{2}$ inclusive, our answer is $\boxed{20}$.

41. The number 9^n ends with 1 if n is even, and ends with 9 if n is odd. Since $8^{7^{6^{5^{4^{3^{2^1}}}}}}$ is even, the last digit of $9^{8^{7^{6^{5^{4^{3^{2^1}}}}}}}$ is $\boxed{1}$.

42. $1.9\overline{8} = \frac{19+0.\overline{8}}{10} = \frac{19+\frac{8}{9}}{10} = \frac{171+8}{90} = \boxed{\frac{179}{90}}$.

43. Since Archimedes' position in the line is fixed, it is enough to find the number of ways to arrange Bernoulli, Cauchy, Descartes, and Euler in a line. Because Descartes and Euler must stand next to each other, we temporarily view them as a single unit. The number of ways to arrange Bernoulli, Cauchy, and the "Descartes-Euler unit" is $3 \times 2 \times 1 = 6$, and the number of ways to arrange Descartes and Euler in the "unit" is 2, so our answer is $6 \times 2 = \boxed{12}$.

44. Trial and error gives that $\boxed{7}$ is the smallest positive integer that cannot be represented as the sum of three not necessarily distinct perfect squares.

45. If the first roll is a 1, we have six choices for the second roll. If it is a 2, that gives us three choices for the second roll (2, 4, 6). If it is a 3, then there are only two choices (3, 6), and for 4, 5, and 6, there is only one choice (itself). That gives us a probability of $\frac{6+3+2+1+1+1}{36} = \boxed{\frac{7}{18}}$.

46. The expression factors to $(x+4)(x-2)$, which can only be a prime number if one of the two expressions is equal to 1 or -1 and the other one is a prime number or a negative number whose absolute value is prime. The only values of x that work are $\boxed{-5}$ and $\boxed{3}$.

47. $((3!)!)! = (6!)! = 720!$; the number of zeroes at the end of 720 is $\lfloor\frac{720}{5}\rfloor + \lfloor\frac{720}{25}\rfloor + \lfloor\frac{720}{125}\rfloor + \lfloor\frac{720}{625}\rfloor = 144+28+5+1 = \boxed{178}$.

48. Let us consider $n \mod 3$. When $n \equiv 0 \mod 3$, $n^2+2n+3 \equiv 0+0+3 \equiv 0 \mod 3$. When $n \equiv 1 \mod 3$, $n^2+2n+3 \equiv 1+2+3 \equiv 0 \mod 3$. Finally, when $n \equiv 2 \mod 3$, $n^2+2n+3 \equiv 4+4+3 \equiv 2 \mod 3$. Thus, $3|n^2+2n+1$ exactly when n is congruent to 0 or 1 mod 3. There are 670 numbers that are $0 \mod 3$, namely $3\cdot 1, 3\cdot 2, \ldots, 3\cdot 670 = 2010$. There are 671 numbers that are $1 \mod 3$, namely $3\cdot 0+1, 3\cdot 1+1, \ldots, 3\cdot 670+1 = 2011$. Thus there are $670+671 = \boxed{1341}$ numbers that satisfy the restrictions.

49. Let a, t, r be the percentage of the apples that Applejack, Twilight Sparkle, and Rainbow Dash can pick in one day, respectively. We are given that $3(a+t) = 4(a+r) = 6(t+r) = 1$, so $a+t = \frac{1}{3}$, $a+r = \frac{1}{4}$, and $t+r = \frac{1}{6}$. Adding these up and dividing by 2 gives $a+r+t = \frac{3}{8}$, so it will take $\frac{1}{a+r+t} = \boxed{\frac{8}{3}}$ days for them to pick all the apples.

50. The region in question is a right triangular pyramid with legs of length 12, 15, and 20 (since its vertices are $(0,0,0), (0,0,12), (0,15,0)$, and $(20,0,0)$. The volume of this pyramid is $\frac{12\cdot 15\cdot 20}{6} = \boxed{600}$.