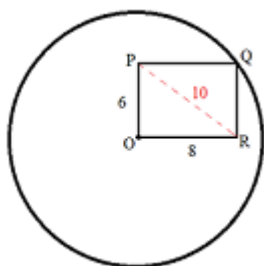


1. There are no A's, 4 E's, 3 I's, 4 O's, and 3 U's, for a total of $\boxed{14}$ vowels.
2. The answer is $\boxed{89}$.
3. The sum of one-eighth and one-eighth is $\frac{1}{8} + \frac{1}{8} = \frac{1}{4}$. Hence, the answer is $\frac{1}{8} \cdot \frac{1}{4} = \boxed{\frac{1}{32}}$.
4. The question is asking for the 13th smallest positive multiple of 3, which is $\boxed{39}$.
5. Since 1 ft is 12 in, Danny Kim is $5 \cdot 12 + 3 = 63$ in tall. He wants to grow until he is 2 in taller than Jeremy Lin, which is 77 in. Therefore, he must grow $77 - 63 = \boxed{14}$ in.
6. The answer is $\boxed{12321}$.
7. The answer is $\frac{60 + 95 + 94}{3} = \boxed{83}$ points.
8. Since Kelley counts one more red hat than JP, it must be that Kelley is wearing a blue hat, whereas JP is wearing a red hat. Therefore, there are total of 1451 people in the room wearing a red hat. Hence, the number of people wearing a blue hat is $2012 - 1451 = \boxed{561}$.
9. The 50th positive even number is 100, and 50th positive odd number is 99, for a sum of $\boxed{199}$.
10. Because Ryan does not finish in the bottom 2, he cannot be 4th. Since Kevin and Claire place higher than Dominic, they cannot be 4th either. Thus, the answer is $\boxed{\text{Dominic}}$.
11. The answer is $\boxed{16.3}$.
12. Note that if we bought 8 slices of pizza individually, it would cost $1.5 \cdot 8 = 12$ dollars per pie. Therefore, Dr. Abramson can save $12 - 9 = 3$ dollars by buying pies instead of slices. Since 160 slices of pizza correspond to $160 \div 8 = 20$ pies, he can save $20 \cdot 3 = \boxed{60}$ dollars.
13. 446 can be factored as the product of 2 and 223, both of which are primes. Hence, the answer is $2 + 223 = \boxed{225}$.
14. Let us group each two consecutive numbers, as following: $(0-1)+(2-3)+(4-5)+\dots+(98-99)$. Each group comes out to -1 , and since there are 50 groups, the answer is $-1 \cdot 50 = \boxed{-50}$.
15. Clearly, the total number of snacks has to be multiple of all of 4, 5, and 6. The smallest such number is $\boxed{60}$.
16. Total time for the practice is $10 + 2 \cdot 30 + 15 = 85$ minutes. Because the practice ended at 6:00 PM, it started at $\boxed{4 : 35 \text{ PM}}$.
17. The perimeter of the equilateral triangle with side length 16 is 48. Because a square consists of 4 congruent sides, each side must have length of 12. Finding the area of this square, we get $12 \cdot 12 = \boxed{144}$.
18. The mean is the arithmetic average of all the elements, median is the middle element of the set, and mode is the element that occurs most frequently. Thus, the mean is 4, median is 4, and mode is 6. Therefore, the answer is $4 + 4 + 6 = \boxed{14}$.
19. From the divisibility rule for 3, we know that a number is divisible by 3 if and only if its sum of digits is divisible by 3. Thus, we must have that $5 + X + 5 + 4$ is a multiple of 3. The largest such digit is $\boxed{7}$.

20. Wooseok has a $\frac{1}{2}$ chance of getting each question right by guessing. Since there are 6 questions, the probability that he gets them all right is $\left(\frac{1}{2}\right)^6 = \boxed{\frac{1}{64}}$.
21. The number of people in marching band must be both multiple of 7 and a square. The only number between 150 and 200 satisfying these properties is $7 \cdot 28 = 14^2 = \boxed{196}$.
22. Dividing both sides of $36a + 30b = 360$ by 180, we get $\frac{36a}{180} + \frac{30b}{180} = \frac{360}{180} \implies \frac{a}{5} + \frac{b}{6} = \boxed{2}$.
23. Note that $17 + 18 + \dots + 45 + 46 = \frac{(17 + 46) \cdot 30}{2} = 945$. Thus, the answer is $47 + 48 + \dots + 75 + 76 = \frac{(47 + 76) \cdot 30}{2} = \boxed{1845}$.
24. There are total of $6 \cdot 6 = 36$ possibilities for results of rolling two dice. Among those possibilities, we obtain a sum of 5 for 4 cases: 1 and 4, 2 and 3, 3 and 2, and 4 and 1. Hence, the answer is $\frac{4}{36} = \boxed{\frac{1}{9}}$.
25. The answer is $\frac{1}{4} \cdot \frac{1}{5} + \frac{3}{4} \cdot \frac{1}{12} = \boxed{\frac{9}{80}}$.
26. Observe that $1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19 = 100$, whence there are $\boxed{10}$ rows of cans.
27. Let the numerator and denominator of the unknown fraction be n and d , respectively. Then, $\frac{n}{d} = \frac{n + 48}{d + 64}$. By cross-multiplying and simplifying, we obtain $64n = 48d$, giving $\frac{n}{d} = \frac{48}{64} = \boxed{\frac{3}{4}}$.
28. Note that the smallest such number is 11111, while the largest is 99999, and their sum is 111110. Also, the second smallest number is 11113, while the second largest is 99997, and their sum is 111110. As such, we can pair the n th smallest number and n th largest number, to get a sum of 111110, and an average of 55555. Further noting that the number exactly in the middle is 55555, the answer is $\boxed{55555}$.
29. There are $5 \cdot 4 \div 2 = 10$ ways to pick a pair of boys, and since each pair plays two games, there are 20 games played between two boys. Similarly, there are $2 \cdot 3 \div 2 \cdot 2 = 6$ games played between two girls. Furthermore, since each of the three girls plays one game with each of the five boys, there are $3 \cdot 5 = 15$ games played between boys and girls. Adding, we count a total of $20 + 6 + 15 = \boxed{41}$ games at the tournament.
30. 2500 can be expressed as $5^4 \cdot 2^2$. Thus, it has $(4 + 1)(2 + 1) = 15$ divisors total, and the only ones that are not multiples of 5 are $5^0 \cdot 2^0, 5^0 \cdot 2^1$, and $5^0 \cdot 2^2$. Thus, there are $15 - 3 = \boxed{12}$ divisors that are multiples of 5.
31. Average damage or the expected value of the damage can be calculated by multiplying the probability of a result by its result, and then summing all of these products together. Hence, the answer is $0.9 \cdot 100 + 0.1 \cdot 200 = \boxed{110}$.

32. There is 1 case where the two cats are on the ends and all of the dogs are in the middle. For the cases where there are two dogs on the ends, we must arrange one leftover dog and two cats in the three center spots. This can be done in 3 ways. Thus, there are 4 total arrangements.

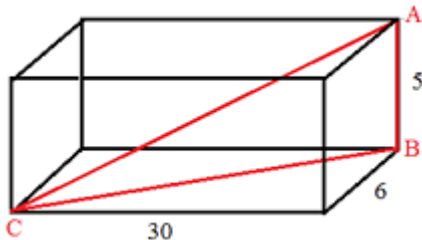


33. Consider the diagram above. Because the lengths of the diagonals of a rectangle are equal, we can find the radius OQ by calculating the length of PR . Using the Pythagorean Theorem, we obtain $PR = \sqrt{6^2 + 8^2} = 10$. The area of the circle is then $\pi \cdot 10^2 = \boxed{100\pi}$.
34. Here, we use the fact that work rate multiplied by the time spent working equals the amount of work done. Call Paula's work rate per minute p and Abhiram's work rate a . We are given that $40(p + a) = 60p$, which simplifies to $40a = 20p$. If we multiply both sides by 3, we obtain $120a = 60p$. Note that $60p$ is the work needed to finish painting the house, since Paula takes 60 minutes to do so with work rate p . Hence, Abhiram needs $\boxed{120}$ minutes to paint the house alone.
35. The first ten "cycles" of the pattern cover the first $1 + 2 + \dots + 10 = 55$ terms, and the eleventh cycle is incomplete. So let's first consider the sum of the terms in the first ten cycles. Of these, 10 include the term one, 9 include the term two, 8 include the term three, and so on. So the sum of the first 55 terms can be simplified as $10 \cdot 1 + 9 \cdot 2 + 8 \cdot 3 + \dots + 3 \cdot 8 + 2 \cdot 9 + 1 \cdot 10 = 220$. Now we can add the remaining eight terms of the incomplete 11th cycle of the pattern. Thus, the answer is $220 + 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 = \boxed{256}$.
36. We know, from 1 across and 1 down, that a two-digit perfect cube and a two-digit perfect square must have the same tens digit. The only options for the tens digit are therefore 2 (27 and 25) or 6 (64 and 64). However, if the tens digit is 6, then by 3 across we must have a multiple of 13 that begins with a 4. However, no such multiple exists, so the tens digit cannot be 6. Instead, it must be 2. We are now looking for a two-digit multiple of 13 that begins with a 5, and a two-digit multiple of 8 that begins with 7. We see that putting 2 in the last two boxes, giving 72 and 52, satisfies the criteria. Our final crossword looks like this:

| | |
|---|---|
| 2 | 7 |
| 5 | 2 |

37. See # 36
38. See # 36
39. See # 36

40. Let b and g be the number of boys and girls at BCA, respectively. Then, the condition in the problem gives $\frac{3}{4}b + \frac{2}{5}g = \frac{1}{2}(b + g)$. Multiplying both sides of the equality by 20, we get $15b + 8g = 10b + 10g \implies 5b = 2g$. Hence, the ratio of boys to girls at BCA is $\frac{b}{g} = \boxed{\frac{2}{5}}$.



41. In the diagram above, the longest segment that can be placed fully inside the prism is the space diagonal AC . Use the Pythagorean Theorem to find $BC = \sqrt{6^2 + 30^2} = \sqrt{936}$. Use it again to find that $AC = \sqrt{936 + 5^2} = \sqrt{961} = \boxed{31}$.
42. Note that the three sides of the triangle satisfy the Pythagorean Theorem, implying that it is a right triangle. Thus, the diameter of the circle is 20, whence its circumference is $\boxed{20\pi}$.
43. Call the two happy pair numbers x and y , where $x > y$. Then $x + y = 5823$, and $x - y = 1111$. Adding the equations of this system and then dividing by two yields the value of x , which is $\boxed{3467}$.
44. For the sake of argument, let us suppose that the distance between his house and his school is 240 miles. If we let t be the time in hours that Dennis took to get to school, then he drove $\frac{t}{2}$ for 40 mph and $\frac{t}{2}$ for 60 mph. Since distance is product of time and velocity, we get that $240 = \frac{t}{2} \cdot 40 + \frac{t}{2} \cdot 60 = 50t \implies t = 4.8$. Thus, it took him 4.8 hours to get to school. Secondly, on the way home, Dennis drove 120 miles with 40 mph, and another 120 miles with 60 mph. Hence, it took him $\frac{120}{40} + \frac{120}{60} = 5$ hours to get back home. Finally, since he took $5 + 4.8 = 9.8$ hours to drive a total of $2 \cdot 240 = 480$ miles, his average speed for the entire trip is $\frac{480}{9.8} = \boxed{\frac{2400}{49}}$ mph.
45. Note that from -12 to 15 , there are 8 positive odd integers, 7 positive even integers, 6 negative odd integers, and 6 negative even integers. We divide the problem into four cases, depending on which of those categories the integer first selected falls into. If the first integer is positive and odd, the second must be positive and even, and this happens with probability $\frac{8}{28} \cdot \frac{7}{27}$. If the first integer is positive and even, the second can be any positive integer, and this happens with probability $\frac{7}{28} \cdot \frac{14}{27}$. If the first integer is negative and odd, the second must be negative and even, and this happens with probability $\frac{6}{28} \cdot \frac{6}{27}$. If the first integer is negative and even, the second can be any negative integer, and this happens with probability $\frac{6}{28} \cdot \frac{11}{27}$. Adding all of these probabilities, the answer is $\boxed{\frac{64}{189}}$.

46. Starting from the powers of six that have at least two digits: 6^2 ends in 36, 6^3 ends in 16, 6^4 ends in 96, 6^5 ends in 76, and 6^6 ends in 56. Since 6^7 ends in 36, we see that the pattern repeats itself every cycle of five starting from 6^2 . We thus consider the remainder of $34 - 2 = 32$ when it is divided by five, and see that like 6^4 , the 34th power of six ends in $\boxed{96}$.
47. The probability that Izzy wins on her first turn is $\frac{2}{3}$. In order for her to win in her second turn, both her and Arthur must get tails on their first turns, and Izzy has to get heads on her second turn, and this happens with probability $\left(\frac{1}{3}\right)^2 \cdot \frac{2}{3}$. In order for her to win in her third turn, all of four previous flips must be tails, and Izzy has to get heads on her third turn, and this happens with probability $\left(\frac{1}{3}\right)^4 \cdot \frac{2}{3}$, and so on. Hence, we need to evaluate the infinite sum $S = \frac{2}{3} + \left(\frac{1}{3}\right)^2 \cdot \frac{2}{3} + \left(\frac{1}{3}\right)^4 \cdot \frac{2}{3} + \dots$. Multiplying both sides of the expression by 9, we obtain $9S = 6 + \frac{2}{3} + \left(\frac{1}{3}\right)^2 \cdot \frac{2}{3} + \left(\frac{1}{3}\right)^4 \cdot \frac{2}{3} + \dots$. Subtracting S from $9S$, we get $8S = 6$, from which it follows that the answer is $\boxed{\frac{3}{4}}$.
48. The triangular faces contribute $3 \cdot 8 = 24$ edges. The octagonal faces contribute $8 \cdot 6 = 48$ edges. However, each edge is counted twice since every edge is the meeting of two faces. Therefore, there are $\frac{24 + 48}{2} = \boxed{36}$ edges in total.
49. Because the bug moves to either adjacent vertex with equal probability, the bug has equal probability of taking any of following path: ABAB, ABAC, ABCA, ABCB, ACAB, ACAC, ACBA, ACBC. The bug visits all 3 vertices in exactly 6 of these paths, so the answer is $\frac{6}{8} = \boxed{\frac{3}{4}}$.
50. Since a diagonal in a square bisects any of the four right angles, $\angle ECG = 45^\circ$, and thus $\triangle ECG$ is an isosceles right triangle. Hence, $EF = 2 \cdot EG = 2 \cdot CG = 2(12 - GD) = 2(12 - EF)$. Thus, we obtain that $EF = 8$, and so $CG = 4$. Then, the area of $\triangle AEC = \frac{AC \cdot CG}{2} = \frac{12 \cdot 4}{2} = \boxed{24}$.