1. There are $\boxed{31}$ letters in this question.

2. 14 quarters is $14 \cdot 25 = 350$ cents. Divide this by 5 cents per nickel $\frac{350}{5} = \boxed{70}$.

3. 100 ninth graders $\times$ 9 hibernation days per ninth grader = 900 hibernation days. 100 tenth graders $\times$ 10 hibernation days per tenth grader = 1000 hibernation days. 100 eleventh graders $\times$ 11 hibernation per eleventh grader = 1100 hibernation days. 100 eleventh graders $\times$ 9 hibernation per eleventh grader = 1200 hibernation days. Thus, the answer is $900 + 1000 + 1100 + 1200 = \boxed{4200}$ hibernation days.

4. $\frac{3}{7} + \frac{7}{3} = \frac{9}{21} + \frac{49}{21} = \boxed{\frac{58}{21}}$.

5. Let K and D be the kitten's and the dog's distance, respectively, from the kitten's starting point. Let T be the time in seconds. The kitten moves at a rate of 12 feet per second, so $K = 12T$ The dog starts 108 feet ahead of the kitten and moves at 9 feet per second, so $D = 9T + 108$ We want T when the kitten and the dog are at the same point, or when $K = D$. $12T = K = D = 9T + 108$ $12T = 9T + 108 \Longrightarrow 3T = 108 \Longrightarrow T = 36$ The kitten will take $\boxed{36}$ seconds to catch the dog.

6. We note that $10^6 = 2^6 \cdot 5^6$ and cancelling 2^6 and 5^6 from the numerator and denominator gives us a final answer of $\boxed{5}$.

7. $(-1)^n = 1$ when n is even, because all of the negatives cancel out with each other. $(-1)^n = -1$ when n is odd, because a negative still remains after cancellation. Knowing that, the question looks as if it depends on whether x is even or odd, but it actually does not. When x is even, $x + 101$ is odd. When x is odd, $x + 101$ is even. Thus, when one of $(-1)^x$ and $(-1)^{x+101}$ is equal to 1,the other is equal to -1. The two terms cancel, and so the answer is $\boxed{0}$.

8. According to the divisibility rule by 9, we know that a number's remainder when divided by 9 is simply the remainder of sum of digits of the number when ti is divided by 9. Hence, the answer is $(1 + 1 + 2 + 3 + 5 + 8 + 1 + 3 + 2 + 1 + 3 + 4) \div 9 = 34 \div 9 = 3 \cdots \boxed{7}$.

9. We multiply both sides of the equation by 28 to get $x = \frac{28 \cdot 36}{84} = \frac{36}{3} = \boxed{12}$.

10. Kevin and Claire each cannot be 4^{th} because they both place higher than someone and Ryan does not finish in the bottom 2 so by process of elimination $\boxed{\text{Dominic}}$ must be 4^{th}.

11. Kelley sees 2010 people plus JP, while JP sees 2010 people plus Kelley. As Kelley and JP see the same 2010 people, the difference in the number of red hats they see must come from one of themselves wearing a red hat and the other not wearing a red hat. Kelley sees more red hats, so that means JP is wearing the red hat and Kelley is not, because Kelley can see JP while JP cannot see himself. That means JP and the 1450 people JP sees wearing red hats are the only people wearing the red hats. There are thus 1451 people wearing red hats. 2012 total people-1451 people wearing red hats=$\boxed{561}$ wearing blue hats

12. An equilateral triangle of side length 12 has a perimeter of $12 \cdot 3 = 36$. A square of perimeter 36 has side length $\frac{36}{4} = 9$ and area $9^2 = \boxed{81}$.

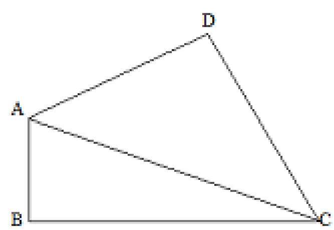
13. $24a + 20b = 240$. We divide both sides by 120 to get $\frac{24a}{120} + \frac{20b}{120} = \frac{240}{120} \Longrightarrow \frac{a}{5} + \frac{b}{6} = \boxed{2}$

14. The number of students in Mr. Lemma's marching band must be both a perfect square and divisible by 6. We are also given that it is between 100 and 150. Of the perfect squares in this range, 121 and 144, only $\boxed{144}$ is divisible by 6.

15. In every question, there are only two answer choices, so he has a $\frac{1}{2}$ chance of guessing a certain question correctly. Since there are 8 questions, and all of the probabilities are independent, the probability of him guessing eight questions correctly is $\left(\frac{1}{2}\right)^8 = \boxed{\frac{1}{256}}$

16. The possible dimensions for a rectangle of perimeter 30 are, after removing ones that are congruent to others already listed, 1×14, 2×13, 3×12, 4×11, 5×10, 6×9 and 7×8. Of these 7×8 produces the highest area of $\boxed{56}$.

17. The mean of the four numbers $3, 7, 5$, and $2x$ is $\frac{3+7+5+2x}{4} = x \Longrightarrow \frac{15+2x}{4} = x$. Solving for x, we get $15 + 2x = 4x \Longrightarrow 15 = 2x \Longrightarrow x = \boxed{\frac{15}{2}}$



18. The area of $ABCD$ is simply the sum of areas of triangles ABC and CDA. Calculating, we obtain $\frac{7 \cdot 24}{2} + \frac{20 \cdot 15}{2} = \boxed{234}$.

19. We will break this up into two cases. First, consider the probability of rolling at least one 2 or 5. By complementary probability, the probability of <u>not</u> rolling either a 2 or a 5 is $\frac{4}{9} \cdot \frac{4}{9} = \frac{16}{36} = \frac{4}{9}$. Thus, the probability of rolling a 2 or a 5 is $1 - \frac{4}{9} = \frac{5}{9}$. However, we must also subtract the two cases when we roll <u>both</u> a 2 and a 5, which happens with probability $\frac{2}{36} = \frac{1}{18}$. Therefore, the probability of rolling a 2 or a 5, but not both, is $\frac{5}{9} - \frac{1}{18} = \boxed{\frac{1}{2}}$.

20. We know, from 1 across and 1 down, that a two-digit perfect cube and a two-digit perfect square must have the same tens digit. The only options for the tens digit are therefore 2 (27 and 25) or 6 (64 and 64). However, if the tens digit is 6, then by 3 across we must have a multiple of 13 that begins with a 4. However, no such multiple exists, so the tens digit cannot be 6. Instead, it must be 2. We are now looking for a two-digit multiple of 13 that begins with a 5, and a two-digit multiple of 8 that begins with 7. We see that putting 2 in the last two boxes, giving 72 and 52, satisfies the criteria. Our final crossword looks like this:

2	7
5	2

21. See #20
22. See #20
23. See #20
24. Let us say that each painter paints x houses each day. We have $3 \cdot 4 \cdot x = 10$, so $x = \frac{10}{12} = \frac{5}{6}$. Let n be the number of painters required to paint 85 houses in 17 days: $n \cdot \frac{5}{6} \cdot 17 = 85$, so $n = \frac{85}{17} \cdot \frac{6}{5} = \boxed{6}$.
25. A shortcut to this problem is realizing that $a\#b = ab - a = a(b-1)$.Thus, $a\#1 = a \cdot 0 = 0$. Since the last step of the problem is computing $x\#1$ where $x = (((25\#16)\#9)\#4)$, our answer is $\boxed{0}$.
26. First we prime factorize $4^6 = 2^{12}$. Letting $a = 2^x$, we have $2^{\mathrm{x} \cdot \mathrm{b}} = 2^{12}$, $x \cdot b = 12$ where b is a positive integer. This gives us $b \in (1,\ 2,\ 3,\ 4,\ 6,\ 12)$. We now check that each of these give a positive integer value for a, which they all do, so we have $\boxed{6}$ ordered pairs.
27. Shocking Thunder has a 10% chance of doing 200 damage and a 90% chance of doing 100 damage. Thus the average damage is $\frac{10}{100} \cdot 200 + \frac{90}{100} \cdot 100 = 20 + 90 = \boxed{110}$.
28. We want to maximize a^b and minimize cd. To maximize a^b we need to make it as positive as possible. This happens when we let $a = 3$ (or -3) and $b = 2$. Next, to minimize cd, we need to make it as negative as possible, so we want to choose the largest combination of a positive number and a negative number. Since we are already choosing one 3 (for a), the most negative product possible is -6, which happens when $c = 3$ and $b = -2$. Consequently, we get that $a = -3$, and so our final answer is $(-3)^2 - (3)(-2) = \boxed{15}$.
29. Let us denote each car type by the number of spaces it takes up. Sedans=1, SUV=2, semitrucks=4. In essence, we are finding the number of ways to make 10 with 1's, 2',s and 4's. With no semitruck's we have 6 ways, namely: 1,1,1,1,1,1,1,1,1,1,2,1,1,1,1,1,1,1,1,2,2,1,1,1,1,1,1, 2,2,2,1,1,1,1, 2,2,2,2,1,1, 2,2,2,2,2. With 1 semitruck: 4,1,1,1,1,1,1, 4,2,1,1,1,1, 4,2,2,1,1, 4,2,2,2, there are 4 ways. With 2 semitrucks: 4,4,1,1, 4,4,2, there are 2 ways. In all, there are 6+4+2=$\boxed{12}$ ways.
30. Since a diagonal in a square bisects any of the four right angles, $\angle ECG = 45^\circ$, and thus $\triangle ECG$ is an isosceles right triangle. Hence, $EF = 2 \cdot EG = 2 \cdot CG = 2(12 - GD) = 2(12 - EF)$. Thus, we obtain that $EF = 8$, and so $CG = 4$. Then, the area of $\triangle AEC = \frac{AC \cdot CG}{2} = \frac{12 \cdot 4}{2} = \boxed{24}$.
31. First, we know that any even number of stamps can be sorted in groups of 2. Then for every odd number greater than or equal to 5, we can sort them by using one group of 5 and then sort the remaining into groups of 2. Therefore, the biggest number of stamps that cannot be sorted is the largest odd number less than 5, or $\boxed{3}$.
32. We begin by finding how many factors of 2 are in 2012: $2012 = 1006 \cdot 2 = 503 \cdot 2^2$. Now we have $2012^{2012} = (503 \cdot 2^2)^{2012} = 503^{2012} \cdot \boxed{2^{4024}}$. (4024 is also acceptable)

33. The minimum number cannot be 2 colors, because 3 edges meet at every vertex, which forces at least 2 of the edges to be the same color. If the three sets of parallel edges in the cube are colored the same color, then no two edges with the same color will meet at a vertex, so our answer is $\boxed{3}$.

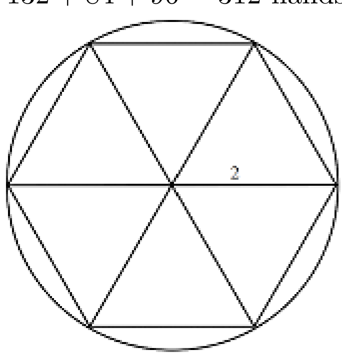
34. Let a be the fraction of girls in BCA and b be the fraction of boys. We have $a+b=1$ and $0.4 \cdot a + 0.75 \cdot b = 0.5$. Subtracting 0.4 times the first equation from the second we have $0.35b = .1$, $b = \frac{2}{7}$. Therefore, $\frac{2}{7}$ of the students in BCA are boys. Substituting for b in the first equation we get $a + \frac{2}{7} = 1$, $a = \frac{5}{7}$. This gives us a ratio of $\frac{\frac{2}{7}}{\frac{5}{7}} = \boxed{\frac{2}{5}}$.

35. Minor arc CBD of circle B and minor arc BAD of circle A are not included in the perimeter and are 120 degree sectors each. Thus, the perimeter consists of $\frac{2}{3}$ of the perimeter of two full circles of radius 3. Therefore, the answer is $\frac{2}{3} \cdot 6\pi \cdot 2 = \boxed{8\pi}$

36. Squaring both sides we have $x^2 + 8x + 16 = x + 6$, or $x^2 + 7x + 10 = 0$. Factoring this (or using the quadratic formula) into $(x+2)(x+5) = 0$ gives us $x = -2, -5$ but $x = -5$ does not work in our original equation (since $\sqrt{x+4}$ is then negative) so $x = \boxed{-2}$ is our only solution.

37. Let the first person get any 2 questions right. The probability the second person does <u>not</u> get either of those questions right is the probability that he gets 2 of the remaining 3 questions right. Since he can do this in $\binom{3}{2} = 3$ ways, and there are $\binom{5}{2} = 10$ ways to get two problems right in general, this probability is $\frac{3}{10}$. Therefore, the probability that they share at least one correct answer in common is $1 - \frac{3}{10} = \boxed{\frac{7}{10}}$

38. There are $\binom{12}{2}$ ways to select two male teachers resulting in $66 \cdot 2 = 132$ handshakes in this category. Similarly there are $\binom{8}{2} \cdot 3 = 84$ handshakes between two female teachers. Lastly there are $12 \cdot 8 = 96$ ways to select one male and one female teacher. This gives us a total of $132 + 84 + 96 = 312$ handshakes.



39. We can break up the hexagon into 6 congruent, equilateral triangles, as shown in the diagram below. Since the area of an equilateral triangle is $\frac{s^2\sqrt{3}}{4}$, the area of six of these triangles is

$\frac{3s^2\sqrt{3}}{2}$. Since the side length of a triangle is the radius of the circle (which is 2), the area is $\boxed{\frac{3\sqrt{3}}{2}}$.

40. For a positive integer of the form $p_1^{e_1} \dots p_n^{e_n}$, the number of factors it has is $(e_1 + 1) \dots (e_n + 1)$. Therefore, for the number to have exactly 12 factors, the expression $(e_1 + 1) \dots (e_n + 1)$ must equal 12. Prime factoring 12, we see that we can satisfy this condition in many ways:

    Case 1: The number has only one factor, i.e. it is of the form p^{e_1}. Then, in order for it to have 12 factors, we must have $e_1 + 1 = 12 \rightarrow e_1 = 11$. Therefore, it must be of the form p^{11}. The smallest prime p is 2, so the smallest number for this case is $2^{11} = 2048$.

    Case 2: The number has two factors, i.e. it is of the form $p_1^{e_1} p_2^{e_2}$. Then, for this to have 12 factors, we must have $(e_1 + 1)(e_2 + 1) = 12$. This can be done in two ways: either $p_1^3 p_2^2$ or $p_1^5 p_2^1$. Either way, we will choose $p_1 = 2$ and $p_2 = 3$ because those are the two smallest primes, and we want the smallest prime possible for the largest exponent. These two give $8 \cdot 9 = 72$ and $32 \cdot 3 = 96$, respectively.

    Case 3: The number has three factors, i.e. it is of the form $p_1^{e_1} p_2^{e_2} p_3^{e_3}$. Then, for this to have 12 factors, we must have $(e_1 + 1)(e_2 + 1)(e_3 + 1) = 12$. This can only be done in one way: if $e_1 = 2$, $e_2 = 1$, and $e_3 = 1$ (letting any $e_n = 0$ reverts to one of the earlier cases). Therefore, the number must be of the form $p_1^2 p_2^1 p_3^1$. Following the rule of giving the largest exponent the smallest possible prime, we get the smallest possible value of that expression by setting $p_1 = 2$, $p_2 = 3$, and $p_3 = 5$, giving us a final answer of $4 \cdot 3 \cdot 5 = 60$.

    Of the three cases, the smallest possible number with exactly 12 factors is $\boxed{60}$.

41. We will solve this problem by complementary probability—finding the probability that the bug does **not** reach all three vertices in three seconds. There are only two ways the bug can achieve this: either the bug goes (to the following vertices in order) BAB or CAC. These two ways both have the same probability, $\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{8}$. Therefore, the probability that the bug does **not** reach all three vertices is $\frac{1}{8} + \frac{1}{8} = \frac{2}{8} = \frac{1}{4}$, and the probability that he does reach all three vertices is $1 - \frac{1}{4} = \boxed{\frac{3}{4}}$.

42. We first select a position to place the e in (there are 5 possible spots), and then we select letters for each unoccupied position from left to right giving us $5 \cdot 9 \cdot 8 \cdot 7 \cdot 6 = \boxed{15120}$.

43. Let the numbers be 2k − 4, 2k − 2, 2k, 2k + 2, 2k + 4. Multiplying all numbers gives us $(2k - 4)(2k + 4)(2k - 2)(2k + 2)(2k) = 32(k - 2)(k + 2)(k - 1)(k + 1)(k)$. Thus 32 must divide this number. Furthermore, there must be at least one integer out of the five consecutive integers $k - 2, k - 1, k, k + 1, k + 2$ that is divisible by 2, at least one divisible by 3, at least one divisible by 4 and at least one divisible by 5 (for example, take 1, 2, 3, 4, 5). Therefore the largest number that must always be a factor of this is $2^5 \cdot 2 \cdot 3 \cdot 4 \cdot 5 = \boxed{3840}$. We can see that this it cannot be any larger since $2 \cdot 4 \cdot 6 \cdot 8 \cdot 10 = 3840$.)

44. We split this into cases depending on the number of heads JP flipped. The first case JP flips no heads with probability $\frac{1}{8}$ where Michael has a $\frac{15}{16}$ chance of getting more heads than JP for a total chance of $\frac{1}{8} \cdot \frac{15}{16} = \frac{15}{128}$ for this case. The next case JP flips heads once with

probability $\frac{3}{8}$ and Michael has a $\frac{11}{16}$ chance of getting more heads for a total of $\frac{3}{8}\cdot\frac{11}{16}=\frac{33}{128}$. Similarly the next case has a total of $\frac{3}{8}\cdot\frac{5}{16}=\frac{15}{128}$ and the last case has a total of $\frac{1}{8}\cdot\frac{1}{16}=\frac{1}{128}$. Adding these cases together there is a total chance of $\frac{15}{128}+\frac{33}{128}+\frac{15}{128}+\frac{1}{128}=\frac{64}{128}=\boxed{\frac{1}{2}}$.

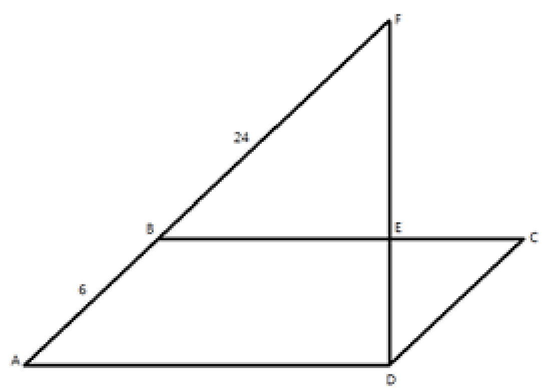
45. The triangular faces contribute $3\cdot 8=24$ edges. The octagonal faces contribute $8\cdot 6=48$ edges. However, each edge is counted twice since every edge is the meeting of two faces. Therefore, there are $\frac{24+48}{2}=\boxed{36}$ edges in total.

46. We observe that $PA^2+PC^2=PB^2+PD^2$ (to see why, draw perpendiculars from P to each side and apply the Pythagorean Theorem to each segment in the equation, taking note of congruent line segments). Substituting the numbers we are given we have $PD=\sqrt{7^2+5^2-8^2}=\boxed{\sqrt{10}}$.

47. This question utilizes Simon' Favorite Factoring Trick, which says $xy+xk+yj+jk=(x+j)(y+k)$. $2ab+8a-3b=36\rightarrow 2ab+8a-3b-12=24\rightarrow(2a-3)(b+4)=24$ So the question now becomes: how many positive integer ordered pairs (a,b) satisfy $(2a-3)(b+4)=24$ Since $2a-3$ is always odd, it is either 1 or 3, so $b+4$ must be 24 or 8, respectively. Thus we have $\boxed{2}$ ordered pairs: $(2,4)$ and $(3,20)$

48. We want to maximize a^b and minimize cd. To maximize a^b we need to make it as positive as possible. This happens when we let $a=3$ (or -3) and $b=2$. Next, to minimize cd, we need to make it as negative as possible, so we want to choose the largest combination of a positive number and a negative number. Since we are already choosing one 3 (for a), the most negative product possible is -6, which happens when $c=3$ and $b=-2$. Consequently, we get that $a=-3$, and so our final answer is $(-3)^2-(3)(-2)=\boxed{15}$.

49. Let the probability Izzy wins be p. Notice that Izzy has two possibilities to win: flip heads on the first turn, which happens with probability $\frac{4}{7}$, or flip tails, and hope Arthur also flips tails, after which Izzy will still have a p chance of winning (since nothing has changed). Therefore, we can write that $p=\frac{4}{7}+\frac{3}{7}\cdot\frac{3}{7}\cdot p$. Solving this equation for p, we get that $p=\boxed{\frac{7}{10}}$



50. Using parallel lines $\overline{AF}$ and $\overline{CD}$, it can be seen that $\triangle CDE$ is similar to $\triangle BFE$ by AAA Similarity. Note that since $ABCD$ is a parallelogram, $CD=AB=6$, and we are given that $BF=24$. Therefore, the ratio of the sides of these two triangles is $24:6=4:1$. The ratio of the areas is the ratio of the sides squared, or $4^2=\boxed{16}$.