

1. By counting, there are  $\boxed{7}$  words in this question.
2. Following the order of operations,  $(1 + 5 \times 6) - 11 = (1 + 30) - 11 = 31 - 11 = \boxed{20}$ .
3. Let's suppose the ice cream man stops on the 1st day in the 12 day period. Since he comes every 5 days, he also stops on the 6th and 11th days. Thus, in the 12 day period, the ice cream man can stop at most  $\boxed{3}$  times.
4. A square has 4 sides, so  $S = 4$ . A pentagon has 5 vertices, so  $P = 5$ . Hence,  $S + P = \boxed{9}$ .
5. Begin by computing  $\frac{1}{8}$  of 64, which is equal to  $64 \div 8 = 8$ . Then find  $\frac{1}{4}$  of this result, which is  $8 \div 4 = 2$ . Finally,  $\frac{1}{2}$  of 2 is simply  $2 \div 2 = \boxed{1}$ .
6.  $A = 2 + 4 + 6 + 8 + 10 = 30$  and  $B = 1 + 3 + 5 + 7 + 9 = 25$ . So,  $A - B = 30 - 25 = \boxed{5}$ .
7. The prime numbers from 1 to 6 are 2, 3, and 5, and there are 6 numbers that can be rolled in total. Hence, the probability is  $\frac{3}{6} = \boxed{\frac{1}{2}}$ .
8. If I am 6 inches taller than Alex, and Arthur is 5 inches taller than I am, then Arthur is a total of  $5 + 6 = \boxed{11}$  inches taller than Alex.
9. The mode is the number that occurs most often in a set, so in the case of  $\{17, 2, 5, 3, 6, 3, 7\}$ , the mode is 3 because it is the only number that appears twice and there are no other numbers that appear more than twice. The median can be found by putting all the numbers in order and finding the one in the middle, which in this case is 5. Therefore, the mean of the median and the mode is  $\frac{3 + 5}{2} = \boxed{4}$ .
10. Since  $2468 = 2 \times 1234$ ,  $\frac{2468}{1234} = 2468 \div 1234 = \boxed{2}$ .
11. There are too many numbers to simply add them all, so instead, we examine the numbers in pairs. The sum of the biggest and the smallest numbers is  $52 + 1 = 53$ , the sum of the second biggest and the second smallest number is  $49 + 4 = 53$ , and so on. Since there are  $\frac{52 - 1}{3} + 1 = 18$  numbers in total, there are  $18 \div 2 = 9$  pairs that each sum to 53. Thus, the total is  $9 \times 53 = \boxed{477}$ .
12. Every number that 36 can be divided by without a remainder is a factor. Thus, 36 has  $\boxed{9}$  factors, namely 1, 2, 3, 4, 6, 9, 12, 18, 36.
13.  $0.2475 = \frac{2475}{10000}$ . Simplifying gives  $\frac{99}{400}$ . Hence,  $a = 99$  and  $b = 400$ . Therefore,  $3a + 4b = 297 + 1600 = \boxed{1897}$ .
14. Following the order of operations,  $\frac{1 + 2 \cdot 3^3}{5} = \frac{1 + 2 \cdot 27}{5} = \frac{1 + 54}{5} = \frac{55}{5} = \boxed{11}$ .
15. To find out how many minutes are in a week, note that there are 60 minutes in an hour, 24 hours in a day, and 7 days in a week. Therefore, a week has  $60 \times 24 \times 7 = 10080$  minutes. Since Arthur can make a friend every 3 minutes, he can make a maximum of  $10080 \div 3 = \boxed{3360}$  friends.
16. Since  $x_n = 1 + \frac{1}{x_{n-1}}$ ,  $x_1 = 1 + \frac{1}{x_0} = \frac{4}{3}$ . By the same reasoning,  $x_2 = 1 + \frac{1}{\frac{4}{3}} = \frac{7}{4}$ ,  $x_3 = \frac{11}{7}$ , and  $x_4 = \boxed{\frac{18}{11}}$ .
17. To make a quarter of the recipe, only  $\frac{1}{4}$  of all the ingredients are needed, so I will need  $\frac{70}{4} = 17.5$  cups. However, since I can only buy crushed rock in whole cup amounts, I will not have enough if I only buy 17 cups, so I will have to buy  $\boxed{18}$ .

18. Let  $F$  be the number of fish that AJ gives to Soonho. After the transfer, AJ has  $9 - F$  fish and Soonho has  $3 + F$  fish. Since AJ has exactly double the amount of fish that Soonho has,  $9 - F = 2(3 + F)$ . Solving this equation for  $F$  gives  $F = \boxed{1}$ .
19. The Pythagorean Theorem says that in any right triangle with legs  $a$  and  $b$  and hypotenuse  $c$ ,  $a^2 + b^2 = c^2$ . Since  $3^2 + 4^2 = 25 = 5^2$ , this is a right triangle with legs 3 and 4. Therefore, its area is  $\frac{3 \times 4}{2} = \boxed{6}$ .
20. Since the triangle is equilateral and the pentagon is regular, every side of this resulting figure will have length 2. Drawing this shows that the figure has 8 sides none of which are overlapping, so the perimeter is  $2 \times 8 = \boxed{16}$ .
21. Number the vertices from 1 to 12 so that vertex A, where the ant starts, is vertex 1. If the ant travels for an odd number of seconds, it can only land on even-numbered vertices - try this out and you will see why! Since 9001 is odd, the ant can never land back on its starting vertex, making the probability  $\boxed{0}$ .
22. To find the greatest common divisor of these two numbers, begin by finding the factors. Note that 1123 is a prime number, so its only divisors are 1 and 1123. 2234 is only divisible by 1, 2, 1117, and 2234. The only divisor these two numbers have in common is  $\boxed{1}$ .

**Alternate Solution :** By the Euclidean algorithm,

$$\begin{aligned} \gcd(1123, 2234) &= \gcd(1123, 2234 - 1123) \\ &= \gcd(1123, 1111) \\ &= \gcd(1123 - 1111, 1111) \\ &= \gcd(12, 1111). \end{aligned}$$

Since  $12 = 2^2 \cdot 3$  and  $2, 3 \nmid 1111$ , we see that the GCD is equal to  $\boxed{1}$ .

23. There cannot be any two-digit numbers containing the digit 0 that are “somewhat similar” because a two-digit number can’t have a 0 in the tens place. However, every other two-digit number can have its digits flipped to result in another two-digit number. One in every 10 two-digit numbers ends in a 0, and there are  $99 - 10 + 1 = 90$  total two-digit numbers, so  $90 \div 10 = 9$  of them have a 0. Thus, there are  $90 - 9 = 81$  that can have their digits flipped. So, if  $m$  is one of these 81 two-digit numbers, then the ordered pair  $(m, n)$ , where  $n$  is the new number gotten by flipping the digits of  $m$ , is “somewhat similar.” Therefore, there are  $\boxed{81}$  total ordered pairs.
24. On the first day of MoneyFest, I get 1 dollar. On the second day, I get  $1 + 2 = 3$  dollars. On the third day, I get  $1 + 2 + 3 = 6$  dollars, etc. Adding all the gains up gives  $1 + 3 + 6 + 10 + 15 + 21 = \boxed{56}$  dollars.
25. Since such a triangle has area 24 and the area of a triangle is given by  $\text{area} = \frac{\text{base} \times \text{height}}{2}$ , the product of the triangle’s legs must be 48. The problem only says that the legs have to have integer lengths, so the hypotenuse does not necessarily have to be an integer. The triangles can therefore have legs of 1 and 48, 2 and 24, 3 and 16, 4 and 12, and 6 and 8. So, there are  $\boxed{5}$  such triangles.
26. The business owner needs \$30 to be the price after 25% has been taken off, so \$30 needs to be 75% of the original price. Thus, the original price has to be  $30 \div 0.75 = \boxed{40}$ .
27. A tetrahedron has 6 edges, so its initial  $E$  is  $6 \times 4 = 24$ . If we cut a tetrahedron of side length 1 from a single vertex, we lose 1 unit from each edge that meets at that vertex, but we also create a side of length 1 unit in between each edge. Therefore, there is no net change in the  $E$  after cutting off a unit tetrahedron from each corner. The  $E$  is still  $\boxed{24}$ .

28. Let the radii of the solids be  $r$  and the height of the solids be  $h$ . Recall that the volume of a cone is  $\frac{1}{3} \cdot \pi r^2 \cdot h$ , while the volume of a cylinder is  $\pi r^2 \cdot h$ . Thus, the cylinder can hold  $\frac{\pi r^2 \cdot h}{\frac{1}{3} \cdot \pi r^2 \cdot h} = \boxed{3}$  times as much water as the cone.
29. Note that Kelvin claims Alex ate the cake. If he is telling the truth, then both Alex and AJ must be lying. Since there is exactly one liar, Kelvin must be telling a lie. Therefore AJ is telling the truth, and Kelvin ate the cake.
30. Say that the hexagon and the triangle have perimeter 6. Note that because the problem does not give a specific perimeter, any number used as the perimeter will work. The hexagon then has side length 1, and the triangle has side length 2. Since the area of an equilateral triangle with side length  $s$  is  $\frac{s^2\sqrt{3}}{4}$ , the area of the equilateral triangle is  $\sqrt{3}$ . The hexagon can be thought of as 6 equilateral triangles with side length 1 put together. The area of one of these triangles is  $\frac{\sqrt{3}}{4}$ . Therefore, the area of the whole hexagon is  $\frac{3\sqrt{3}}{2}$ . The ratio of the area of the triangle to that of the hexagon is then  $\boxed{\frac{3}{2}}$ .
31. Suppose  $a$  and  $b$  are diametrically opposite such that  $a < b$ . Then there are equal amounts of numbers from  $a$  to  $b$  on the circle and from  $b$  to  $a$ . Since there are  $n - 2$  total numbers excluding  $a$  and  $b$ , there must be  $\frac{n - 2}{2}$  numbers from  $a$  to  $b$ . And since the numbers are placed in order, we have that
- $$b - a = \frac{n - 2}{2} + 1 = \frac{n}{2}.$$
- Specifically, since  $a = 17$  and  $b = 38$ , we see that  $b - a = 38 - 17 = 21 = \frac{n}{2}$ , and so  $n = \boxed{42}$ .
32. Note that if  $n$  is in the set, so is  $-n$ , and so the sum of the numbers in the set is equal to 0. Therefore, the average is also  $\boxed{0}$ .
33. Note that in addition to being able to buy kebabs in groups of 3, 5, and 7 from Balabob, customers can also buy kebabs in groups of  $6 = 3 + 3$ . Note that the numbers 5, 6, and 7, are consecutive so by adding 3 to each of these numbers, a customer can buy 8, 9, and 10. By continuously adding three to the new three numbers, one can see that a customer is capable of buying any number of kebabs greater than 4. However, it is impossible to buy 4 kebabs, so our answer is  $\boxed{4}$ .
34. Note that  $\angle ABC = \angle ADB$ . Thus, by similar triangles, we have  $\frac{AC}{BC} = \frac{BC}{CD}$ , and so  $BC^2 = AC \cdot CD$ . Thus,  $12^2 = 5 \cdot CD$ , and  $CD = \boxed{\frac{144}{5}}$ .
35. Dividing 1000 by  $5 \cdot 11$  gives 18 and a remainder.  $18 \cdot 55 = 990 < 1000$ , but 990 is divisible by 3. Therefore, the answer we seek is  $990 - 55 = \boxed{935}$ , as 935 is not divisible by 3.
36. Note that since the wind pushes the dog and the man back the same distance every time, the wind gust does not change the distance between the man and the dog. As a result, the wind gusts can be ignored. When the man starts running at 6 : 00 pm, the dog has already run for 2 hours at a speed of 5 mph for a total of 10 miles. Every hour after 6 : 00 pm, the distance between the man and the dog decreases by 5 miles. This means that the man will take 2 hours to catch up to the dog. As a result, the man and the dog meet at  $\boxed{8:00 \text{ P.M.}}$ .
37. Since the dot on the wheel starts at the bottom of the wheel but ends up at the top, the wheel must have rotated  $0.5 + n$  times during the trip, where  $n$  is a nonnegative integer. Since  $(\# \text{ of rotations}) \times (\text{circumference of wheel}) = \text{distance traveled for a wheel that does not slip}$ , it suffices to find  $n$  such that  $35 < (0.5 + n)(4\pi) < 50$ , since the circumference of a circle is  $\pi \times D$

where  $D$  is the diameter of the circle. The only value for a nonnegative integer  $n$  that satisfies the inequality is  $n = 3$ . Thus, the total distance traveled is  $(0.5 + 3)(4\pi) = \boxed{14\pi}$  meters.

38. If we want to find the probability that event  $A$  happens given that event  $B$  happens, we have to divide the probability of both events happening by the probability of event  $B$  happening. (This is how we can calculate such “conditional” probabilities.) So, in this case, event  $A$  is choosing a student named Jim, while event  $B$  is choosing a student whose name begins with J.

Hence, the chance of  $A$  and  $B$  both occurring is equal to  $\frac{5}{18}$ , since there are 5 students whose name is Jim and Jim starts with J. Also, the chance of  $B$  occurring is equal to  $\frac{11}{18}$ , since there are 11 students whose name begins with J. Therefore, our answer is equal to

$$\frac{\text{Chance of A and B}}{\text{Chance of B}} = \frac{\frac{5}{18}}{\frac{11}{18}} = \boxed{\frac{5}{11}}.$$

Alternatively, note that there are 11 students whose names start with J, 5 of whom are named Jim. Therefore our probability is  $\boxed{\frac{5}{11}}$ .

39. Let the number of kangaroos that do not have a pouch be  $X$  and the number of baby kangaroos be  $B$  and the number of adult kangaroos with pouches be  $P$ . Since there are 2013 kangaroos, we have the equation that  $X + B + P = 2013$ . Since each kangaroo with a pouch has a baby we also know that  $B = P$ , and since there are 9 times as many adult kangaroos without pouches as there are baby kangaroos, we also know that  $X = 9B$ . Thus, rewriting our first equation in terms of  $X$ , we have that  $X + \frac{X}{9} + \frac{X}{9} = 2013$ . Solving for  $X$ , we obtain that  $X = \boxed{1647}$ .

40. To get rid of the absolute value sign, we can say that  $\sqrt{n} - 7 < 1$  or  $\sqrt{n} - 7 > -1$ . For the first equation, we can say that  $\sqrt{n} < 8$ , then square both sides to get that  $n < 64$ . For the second equation, we get that  $\sqrt{n} > 6$ , and square both sides to get that  $n > 36$ . Thus  $n$  can be all integers from 37 to 63. There are  $63 - 37 + 1 = \boxed{27}$  such integers.

41. Dividing, we see that  $\frac{3}{14} = 0.2142857142\dots = 0.2\overline{142857}$ , and so the sequence 1, 4, 2, 8, 5, 7 repeats every 6 digits. The 81st digit will be the 80th digit of this repeating sequence. Note that after the sequence repeats 13 times, the sequence will start again at the  $6 \times 13 + 1 = 79$ th digit of the repeating section, which will be 1. The 80th digit of the repeating section, which is the same as the 81st digit of the whole thing, will then be  $\boxed{4}$ .

42. We can minimize the perimeter of the rectangle by making it a square. If a square has area 100 it has a side length of 10 so its perimeter is 40. To maximize the perimeter of the rectangle, we chose a rectangle with width 1 and length 100, so the rectangle has a perimeter of 202. The difference between these two values is  $202 - 40$  or  $\boxed{162}$ .

43. The diagonal of this rectangle will be the diameter of the circle. Since the side lengths are 6 and 8, by the Pythagorean Theorem, the diameter will be  $\sqrt{6^2 + 8^2} = 10$ . The area of the circle is then  $\pi r^2 = \pi \left(\frac{10}{2}\right)^2 = \boxed{25\pi}$ .

44. If the side lengths of a rectangular prism are  $a, b, c$ , then the diagonal of one face can be expressed as  $\sqrt{a^2 + b^2}$ . The diagonal of the whole prism can then be expressed as  $\sqrt{\left(\sqrt{a^2 + b^2}\right)^2 + c^2} = \sqrt{a^2 + b^2 + c^2} = 13$ . Squaring both sides gives  $a^2 + b^2 + c^2 = 169$ . Since  $a, b, c$  are all integers, we wish to find integer values that work in this equation.

Without loss of generality, we can assume that  $a \leq b \leq c$ . Thus  $a^2 + b^2 + c^2 \leq 3c^2$ , and so  $c^2 \leq 56$ . Thus we need only check  $c \geq 8$ .

- If  $c = 8$ , then  $a^2 + b^2 = 105$ . Hence  $b$  can either be 7 or 8. If  $b = 7$ , then  $a^2 + b^2 \leq 2b^2 = 98 < 105$ , but if  $b = 8$  then  $a = \sqrt{41}$ , which is not an integer. Hence  $c \neq 8$ .
- If  $c = 9$  then  $a^2 + b^2 = 88$ . Another routine check shows that  $b = 8, 9$  return non-integer values for  $a$ .
- If  $c = 10$  then  $a^2 + b^2 = 69$ . Then  $b = 6, 7, 8, 9$  again all fail.
- If  $c = 11$  then  $a^2 + b^2 = 48$ . We again verify that there are no solutions.
- If  $c = 12$  then  $a^2 + b^2 = 25$ , which has solution  $a = 3, b = 4$ .

Thus our volume is  $3 \cdot 4 \cdot 12 = \boxed{144}$ .

**Alternate solution :** Suppose  $a^2 + b^2 + c^2 = 13^2$  as above. We first observe that  $5^2 + 12^2 = 13^2$ . Since we also recall that  $5^2 = 4^2 + 3^2$ , we get that  $13^2 = 12^2 + 4^2 + 3^2$ . Hence the volume is  $3 \cdot 4 \cdot 12 = \boxed{144}$ .

45. When Johnny bought the  $y$  mugs, he spent  $\$10y$  and was  $\$15$  short, meaning that  $10y = x + 15$ . When he put back half the mug, thus spending a total of  $\$5y$ , he had  $\$25$  left over, so  $5y = x - 25$ . Subtracting the second equation from the first gives  $5y = 40 \implies y = 8$ . Plugging this value for  $y$  into the first equation results in  $80 = x + 15 \implies x = 65$ . The sum of  $x$  and  $y$  is therefore  $8 + 65 = \boxed{73}$ .

46. The sum of the columns and rows takes into account every entry twice, so no matter what arrangement the numbers are in, the sum of all the columns and all the rows is  $2(1+2+\dots+9) = 90$ . Thus, the sum of the diagonals is  $124 - 90 = \boxed{34}$ .

47. Let the speed of the turtles (in meters/day) be  $V$ ,  $T$ , and  $S$ , for Victor's turtle, Tony's turtle, and Sam's turtle respectively. From the problem, we know that  $V = 2S$  and  $S = 2T$ . Rewriting  $V, S$ , and  $T$  all in terms of  $T$ , we see that the speeds of the turtles are  $4T$ ,  $2T$ , and  $T$  respectively. Next, since each turtle runs an equal amount of the relay race, each turtle runs 4 meters. Also, rearranging the equation  $\text{distance} = (\text{rate})(\text{time})$  yields  $\text{time} = \frac{\text{distance}}{\text{rate}}$ . It took 14 days for the race to be completed so  $14 = \frac{4}{4T} + \frac{4}{2T} + \frac{4}{T}$ . This simplifies to  $14 = \frac{1}{T} + \frac{2}{T} + \frac{4}{T}$ , which further simplifies to  $14 = \frac{7}{T}$  so  $T = \boxed{\frac{1}{2}}$ .

48. Let  $n = 100a + 10b + c$ , with  $a, b, c$  being digits such that  $1 \leq a \leq 9$  and  $0 \leq b, c \leq 9$ . A three-digit palindrome is any number whose first and last digits are equal, and so if  $n$  is a three-digit palindrome then  $a = c$ . Hence,  $n = 101a + 10b$ . Next, we must also have that  $n$  is divisible by 11. So since

$$n = 101a + 10b = 99a + 2a + 11b - b = 11(9a + b) + 2a - b,$$

and since  $11 \mid 11(9a + b)$ , we must have that  $11 \mid (2a - b)$ . So we need to find the number of ordered pairs  $(a, b)$  such that  $2a - b$  is divisible by 11.

Since  $a < 10$ ,  $2a < 20$ , and so we can only have that  $2a - b = 0$  or  $2a - b = 11$ . If  $2a - b = 0$ , then we get  $(a, b) = (1, 2), (2, 4), (3, 6), (4, 8)$ . If  $2a - b = 11$ , then  $(a, b) = (6, 1), (7, 3), (8, 5), (9, 7)$ . Hence, there are 8 such ordered pairs. And since each one leads to a unique three-digit palindrome, there are  $\boxed{8}$  such numbers.

49. We consider two cases.

- Case 1 is that Arthur finished second. Since James finished before Dennis, the possibilities are James-Arthur-Dennis-Wang, James-Arthur-Wang-Dennis, and Wang-Arthur-James-Dennis, giving a total of 3.
- Case 2 is that Arthur finished third. The possibilities are then James-Dennis-Arthur-Wang, Wang-James-Arthur-Dennis, and James-Wang-Arthur-Dennis, giving a total of 3.

Therefore, there are  $3 + 3 = \boxed{6}$  ways the race could have ended.

50. We can break this problem into cases depending on the number of elements in each subset. We will have 5 different cases since a subset can contain either 1 element, 3 elements, 5 elements, 7 elements, or 9 elements.

**Case 1:** Each subset contains only 1 element There are 10 ways to create a set with only 1 elements.

**Case 2:** Each subset contains 3 elements There are 10 ways to pick the first element, 9 ways to pick the second element and 8 ways to pick the third element. However since order does not matter, we have over counted. Thus we must divide  $10 * 9 * 8$  by the number of ways to arrange 3 elements, which is  $3 * 2 * 1$ . So our answer is  $\frac{10 * 9 * 8}{3 * 2 * 1}$  or 120.

**Case 3:** Each subset contains 5 elements There are 10 ways to pick the first element, 9 ways to pick the second element, 8 ways to pick the third element, 7 ways to pick the fourth, and 6 ways to pick the fifth element. However since order does not matter, we have over counted once again. Thus we must divide  $10 * 9 * 8 * 7 * 6$  by the number of ways to arrange 5 elements, which is  $5 * 4 * 3 * 2 * 1$ . So our answer is  $\frac{10 * 9 * 8 * 7 * 6}{5 * 4 * 3 * 2 * 1}$  or 252.

**Case 4:** Each subset contains 7 elements Note that we can compute this case similar to how we did the previous cases, but we can exploit symmetry to get the answer more quickly. Note that overtime you pick a subset containing 3 elements, you leave behind a subset containing 7 elements. Thus the number of subsets of 7 elements is equal to the number of subsets of 3 elements, which is 120.

**Case 5:** Each subset contains 9 elements By the logic of Case 4, we see that this case has the same number of subsets as Case 1 so this case has 10 subsets. Summing up the sums of all of the 5 cases, we have  $10 + 120 + 252 + 120 + 10$ , which is  $\boxed{512}$ , our final total.

**Remark :** Did you notice that  $512 = \frac{1024}{2} = \frac{2^{10}}{2}$ , and that 10 was the number of elements in our original set? Do you think that this is a coincidence?