Joe Holbrook Memorial Math Competition

4th Grade Solutions

October 15, 2017

1. There are 75 minutes for 40 questions. $75/40 = \boxed{15/8}$
2. $2 \cdot 3 + 5 \cdot 4 + 2 \cdot 5 = 6 + 20 + 10 = \boxed{36}$.
3. It is 2 hours and 36 minutes from 3:00, and as there are 60 minutes in an hour, $2 \cdot 60 + 36 = \boxed{156}$
4. Note there are 60 minutes in an hour and 60 seconds in a minute, so traveling 60 miles would take 60 minutes and $7 \cdot 60 = 420$ seconds, or $60 + 7 = \boxed{67}$ seconds.
5. Since addition and subtraction come before division and multiplication, $2 + 5 \cdot 2 - 5 = (2 + 5) \cdot (2 - 5)$. Now, since operations are performed right to left, $(2 + 5) \cdot (2 - 5) = 7 \cdot 3 = \boxed{21}$.
6. The larger square has side lengths of 6, which means that it has a perimeter of 24. Meanwhile, the smaller square, which has side lengths 3, has perimeter 12. Therefore, their collective perimeter is 36. Since the side of the smaller square is glued to the side of the larger square, you must subtract the edge between them, which is 3 for each square. Therefore, $36 - 6 = \boxed{30}$.
7. We factor 2670: $2670 = 2 \cdot 3 \cdot 5 \cdot 89$, so the largest prime factor is $\boxed{89}$.
8. $20\% \cdot 300 = 60$ of the donuts had sprinkles. Likewise, 30 had chocolate, and 111 had jelly. Thus, there were $300 - 60 - 30 - 111 = \boxed{99}$ plain donuts.
9. The battery will recharge at the rate of $2 - 0.5 = 1.5\%$ per minute. Because it needs to recharge 60%, the answer is $\dfrac{60}{1.5} = \boxed{40}$.
10. There are 1000 meters in a kilometer, so it takes Malik $12.21 \cdot \dfrac{5000}{100} = 12.21 \cdot 50 = \boxed{611.5}$ seconds to finish the race.
11. Every sixth student will have both hands raised. $\dfrac{12}{6} = \boxed{2}$.
12. We bring all the fractions to a common denominator of 60 to get $\dfrac{45}{60} + \dfrac{48}{60} + \dfrac{100}{60} = \boxed{\dfrac{193}{60}}$.
13. Let x be the number of shots he must make to reach his goal. We have the inequality $\dfrac{7 + x}{15 + x} \geq 0.7$, or $x \geq \dfrac{35}{3}$. Since x must be a positive integer, the minimum valid value for x is $\boxed{12}$.
14. It takes Jaylen $\dfrac{2}{3}$ hours, or 40 minutes, to get to the park, while it only takes Derrick $\dfrac{2}{5}$ hours, or 24 minutes. Consequently, Jaylen must leave his house $40 - 24 = \boxed{16}$ minutes earlier than Derrick does.
15. $\dfrac{3}{8}x + \dfrac{1}{3}x + 840 = x$, so $x = 840 \cdot \dfrac{24}{7} = 2880$. Thus, the number of miles with cruise control enabled comes out to $2880 \cdot \dfrac{3}{8} = \boxed{1080}$.
16. Call the original number of boats x. The condition tells us that $6(x + 1) = 9(x - 1)$, which can be solved and $x = 5$. Thus the number of people is $6 \cdot (5 + 1) = \boxed{36}$.
17. We have three choices for literature, 2 choices for history, 4 four math, and 2 for foreign language, for a total of $3 \cdot 2 \cdot 4 \cdot 2 = \boxed{48}$ possibilities.

18. We have to choose the remaining 2 members out of the 5 other people, so the answer is $\binom{5}{2} = \boxed{10}$.

19. Consider the straight segment between the two centers. 3 units of this is contained in one circle, and 5 units is contained in another. Therefore, the shortest distance from two points on each circle is $14 - 5 - 3 = \boxed{6}$.

20. We can write a system of equations where x is the number of octopi and y is the number of crabs. Since each creature has one head, $x + y = 29$. Each octopus has 8 "legs" and each crab has 10 legs so $8x + 10y = 256$. From the first equation, we get $y = 29 - x$. Substituting this into the second equation, we get $8x + 10(29 - x) = 256 \Rightarrow 290 - 2x = 256 \Rightarrow 2x = 34 \Rightarrow x = 17$, so the number of octopi is $\boxed{17}$.

21. There are 31 days in July, so we consider the positive integers less than or equal to 31 that are perfect squares or powers of 2. The perfect squares are $1, 4, 9, 16, 25$, and the powers of 2 are $1, 2, 4, 8, 16$. Thus, Haneul and Julia video-call each other for half an hour on the $1st, 4th, 9th, 16th$, and $25th$, and call for an hour on the $2nd$ and $8th$, so they spend a total of $5 \cdot \frac{1}{2} + 2 \cdot 1 = \boxed{\frac{9}{2}}$ hours communicating.

22. Expressing the information numerically, letting x be the number, yields $39 + x = 2x - 5$, so $x = \boxed{44}$.

23. Let Youjung and Yousun have a, b pieces of candy, respectively. Then $a + b = 48, b - 5 = a + 5$ Systems of equation: $a + b = 48, b - a = 10$. Add 2 equations, $2b = 5 \implies b = 29, a = 19$, so Yousun had $\boxed{29}$ candies.

24. $12 \cdot 1.1 \cdot 0.9 = \boxed{11.88}$. Note that order does not matter in performing this calculation!

25. Note that the units digit of 7^n repeats in cycles of 4. Since $\frac{2017}{4} = 504 + \frac{1}{4}$, the unit digit is the same as $7^1 = \boxed{7}$.

26. Every 7th time Annie mows her lawn is a Sunday, so the 29th time will be a Sunday. Therefore, the 30th time will be a $\boxed{\text{Saturday}}$.

27. The given inequality can be factored as $(n-1)(n-2) \leq 0$. For this to hold, n must satisfy $1 \leq n \leq 2$. There are only two integers in this range, namely $\boxed{1,\ 2}$.

28. There are $\frac{7!}{2!}$ possible ways of rearranging Tiffany as there are 7 letters and the f is repeated twice. There are 6! combinations of rearranging Tifany as there are 6 letters and no repeated letters. $\frac{7!}{2!} = 2520$ and $6! = 720.2520 - 720 = \boxed{1800}$.

29. By the identity $\text{lcm}(a, b) \cdot \gcd(a, b) = ab$, it follows that $ab = 20 \cdot 10 = \boxed{200}$.

30. Note that $0\Delta a = a\Delta 0 = 0$ if $a \neq 0$. Thus $((2\Delta 6)\Delta 3)\Delta(0\Delta(3\Delta 7)) = \boxed{0}$.

31. There are $\binom{6}{3}$ ways to choose 3 socks of the same color, and there are 3 different colors. Meanwhile, there are $\binom{18}{3}$ ways to choose 3 socks. Therefore, the probability is: $3 \cdot \frac{\binom{6}{3}}{\binom{18}{3}} = \boxed{\frac{5}{68}}$.

32. Triangles $\triangle ABD$ and $\triangle BDC$ are equilateral since $\frac{120°}{2} = 60°$ so the total area is $2 * \frac{6^2\sqrt{3}}{4} = \boxed{18\sqrt{3}}$

33. By properties of $30 - 60 - 90$ degree triangles, the height of the tree must be $6\sqrt{3}$ feet tall. As the triangle formed by the position of the bug, David, and the base of the tree forms another $30 - 60 - 90$ degree triangle, the height of the bug above the ground is $\frac{6}{\sqrt{3}} = \boxed{2\sqrt{3}}$ feet.

34. $1200 = 2^4 \cdot 3 \cdot 5^2$, so there are $5 \cdot 2 \cdot 3 = 30$ total factors and $3 \cdot 1 \cdot 2 = 6$ perfect square factors. Therefore, there are $\boxed{24}$ non-perfect square factors of 1200.

35. As $AB = 8$ and $BC = AD = 6$, by the Pythagorean Theorem, it follows that $BD = 10$. As M is the midpoint of BD, DM must be 5. As AN is the height to the base BD in triangle $\triangle ABD$, $\frac{1}{2} \cdot AN \cdot BD = \frac{1}{2} \cdot AB \cdot AD \to AN = \frac{24}{5}$. By the Pythagorean Theorem in triangle $\triangle AND$, $DN^2 = AD^2 - AN^2 \to DN = \frac{18}{5} \to MN = \boxed{\frac{7}{5}}$.

36. Set side BC to be of length x. Then side CD is of length $37.5 - x$, since the sum of the two sides is half the perimeter. Then, $x \cdot 14 = (37.5 - x) \cdot 16$, solving gives us $x = 20$. Thus the area is $20 \cdot 14 = \boxed{280}$

37. Note that $84 = 2^2 \cdot 3 \cdot 7$, $120 = 2^3 \cdot 3 \cdot 5$, and $126 = 2 \cdot 3^2 \cdot 7$. Then, taking the greatest common divisor of each pair of the three values yields $2^2 \cdot 3, 2 \cdot 3 \cdot 7$, and $2 \cdot 3$. Since if an integer divides a corresponding pair of integers, then it must divide the greatest common divisor of the integers, we then sum the total number of divisors of each of the gcds: $3 \cdot 2 = 6, 2 \cdot 2 \cdot 2 = 8$, and $2 \cdot 2 = 4$, so $6 + 8 + 4 = 18$ total factors. However, we are over counting the numbers that divide all three values. Since the gcd of all three is $2 \cdot 3$, we over counted $2 \cdot 2 = 4$ divisors twice, for a total of $2 \cdot 4 = 8$ over counted divisors. Thus, our answer is $18 - 8 = \boxed{10}$.

38. The number of zeros at the end of 2017! is the highest power of 3 dividing 2017!. This equals $\lfloor \frac{2017}{3} \rfloor + \lfloor \frac{2017}{9} \rfloor + \lfloor \frac{2017}{27} \rfloor + \lfloor \frac{2017}{81} \rfloor + \lfloor \frac{2017}{243} \rfloor + \lfloor \frac{2017}{729} \rfloor = \boxed{1004}$.

39. There are 220 ways to choose 3 edges from a cube, which has 12 edges. The 12 edges can be grouped based on their orientations, with each group containing 4 parallel edges. Since the 3 edges have to be pairwise skew, it is imperative to choose one edge from each group. Choose one edge at random from any of the groups. There are 4 possibilities. Two edges from each of the other two groups intersect the selected edge, so choose any of the other two groups and an edge at random. There are 2 possibilities. There is only one edge left from the third group. Thus there are 8 possibilities in total to make the desired 3-edge selection, and the probability is $\boxed{\frac{2}{55}}$.

40. There are $\binom{8}{2}$ ways of choosing where the 0's go. Then, there are $\binom{6}{2}$ ways of choosing where the 1 goes. Now, there are 4 spots left. One of the 2's has to go in the rightmost open spot in order to be pure. Then, there are three open spots left for 2, 7, and 7. There are $\binom{3}{2}$ ways of choosing spots for the 7's. Since these steps are done in order, invoking product rule yields $\binom{8}{2} \cdot \binom{6}{2} \cdot \binom{3}{2} = \boxed{1260}$.