

Joe Holbrook Memorial Math Competition

5th Grade

October 15, 2017

General Rules

- You will have **75 minutes** to solve **40 questions**. Your score is the number of correct answers.
- Only answers recorded on the answer sheet will be graded.
- This is an individual test. Anyone caught communicating with another student will be removed from the exam.
- Scores will be posted on the website. Please do not forget your ID number, as that will be the sole means of identification for the scores.
- You may not use the following aids:
 - Calculator or other computing device
 - Compass
 - Protractor
 - Ruler or straightedge

In addition, you must use the scrap paper supplied by the proctors.

Other Notes

- Write legibly. If the graders cannot read your answer, you will be given no credit for that question.
- Fractions should be written in lowest terms. Please convert all mixed numbers into improper fractions.
- For constants such as e or π , do not approximate your answer: for example, if the answer to a question is 7π , then you should not write 22 or 21.99.
- You do not need to write units in your answers.
- Rationalize all denominators. In addition, numbers within a square root must be squarefree, e.g. $\sqrt{63}$ should be written as $3\sqrt{7}$.
- Ties will be broken by the number of correct responses to questions 31 through 40. Further ties will be broken by the number of correct responses in the last five questions.

1. Youjung loves playing golf. She played total of 9 holes and scored 2 3's, 5 4's, 2 5's. What is her total score?
2. It is currently 12:24pm and Jonathan's mom won't let him go swimming until 3:00pm (on the same day). How many minutes does he have to wait until he can go?
3. Adam is driving a car, and takes 1 minute and 7 seconds to travel 1 mile. How long does it take to travel 60 miles? Express your answer in minutes.
4. If I can spin a rubik cube 6 times per second, and I need to spin it exactly 120 times to solve it, how many seconds does it take for me to solve a rubik cube?
5. In Reverse Land, addition and subtraction come before division and multiplication, and operations are performed from right to left. What is $2 + 5 \cdot 2 - 5$ in Reverse Land?
6. $ABCD$ is a square with side length 6. Let E be the midpoint of BC . If F and G are points outside $ABCD$ such that $EFGC$ is a square, find the perimeter of $ABEFGCD$.
7. What is the largest prime factor of 2670?
8. BCA Math Team loves donuts. At the last practice, there were 300 donuts. 20% of the donuts had sprinkles, 10% of the donuts had chocolate, and 37% of the donuts had jelly. The rest were plain. How many plain donuts were there?
9. Bill's phone has 40% battery left, so he decides to start charging it. The battery charges at 2% per minute if he is not using it. But, he starts using his phone, which discharges it at 0.5% per minute. How long will it take for his battery to charge fully?
10. In the first week of summer, Jordan could only hit his golf ball 120 yards. He reached 144 yards by the second week, and 168 yards by the third week. If Jordan's distance continues to improve by the same amount each week, and summer is a total of 8 weeks, how far (in yards) can he expect to hit his ball by the last week of summer?
11. Jaylen is trying to improve his basketball shooting percentage at the gym, and he refuses to go home until he has made at least 70% of the shots he has taken. So far, he has taken 15 shots and made only 7 of them. Assuming that from now on, he never misses a shot, what is the minimum number of shots that he has to make consecutively to reach his goal?
12. John is playing a game where he starts by doing 1 pushup, and every minute thereafter, does 2 more pushups than he did the previous minute. Once he has done a total of 100 pushups, he stops. If John does his first pushup at 4:15 PM, at what time does he stop?
13. The entire BCA Math Team are trying to cross the Hackensack River and we have a bunch of boats. We realize that if we have another boat, there will be exactly 6 people on each boat; if we take away a boat, there will be exactly 9 people on each boat. How many people are on the team?
14. Wanye the Toed is creating the Council of Toed from him and his friends, Peiyu the Pig, Greggry the Goose, Sampai the Snail, Daniel the Stewart, and Andrew the Antelope. The Council should have 3 members, and Wanye has to be in it. How many choices are there for the Council?
15. Two circles have radius 3 and 5. Their centers are 14 apart. A line segment connects one point on one circle to another point on the other circle. What is the minimum length of all possible line segments?
16. If you flip 1000 coins, there is an $n\%$ chance that at least 500 will land the same way. Find n .
17. What is the unit digits of 2017^{2017} ?
18. My teacher told me I had a 60% chance of getting a homework pass if I got an A+ on my report card, and a 20% chance otherwise. I know I have a 60% chance of getting an A+ on my report card. What is the probability I will get a homework pass?
19. Find all integers n which satisfy the inequality $n^2 - 3n + 2 \leq 0$.
20. An evil goat spelled the name *Tiffany* wrong. The goat had written her name as *Tiffany*. How many more ways are there to rearrange the letters of *Tiffany* than *Tiffany*?
21. Let $x \neq y$, and the sequences x, A, B, C, y and D, x, E, F, y, G are both arithmetic sequences. Find $\frac{G - F}{C - B}$.

22. What is the maximum number of intersections between 17 circles and 17 lines?
23. Let a and b be two distinct real numbers. If the function $f(x) = x^2 + ax + b$ satisfies $f(a) = f(b)$, find the value of $f(2)$.
24. Rhombus $ABCD$ has side length 6 and $\angle ABC = 120^\circ$. What is the area of rhombus $ABCD$?
25. The difference of two 3-digit numbers is 894. Find the product of the digits of the 2 numbers.
26. What is the area of regular octagon with side length $\sqrt{2}$?
27. David is standing 6 feet away from a tree. He sees the top of the tree at an angle of 60 degrees to the ground. If David spots a bug crawling along the tree at an angle of 30 degrees to the ground, how far above the ground was the bug at that instant? (Note: David's height is negligible)
28. A coin is flipped 5 times. What is the probability that at least three consecutive heads are flipped?
29. Triangle ABC has circumcircle Ω with center O and radius 2. If line BO meets Ω again at a point P other than A with $PC = 3$, find the length of side BC .
30. Let $ABCD$ be a rectangle with $AB = 8$ and $BC = 6$. If M is the midpoint of BD and N is the foot of the perpendicular from A to BD , find MN .
31. The Fibonacci sequence is a sequence of numbers where any term after the second is the sum of the previous two terms. Among the first 1000 terms of the sequence, how many are multiples of 11? The first 8 terms are given here: 1, 1, 2, 3, 5, 8, 13, 21.
32. A regular hexagon is inscribed in a circle. Two points are randomly chosen on the circle, and a line segment is drawn between them. What is the probability that this line segment does not intersect the hexagon?
33. When written in base 15, how many terminating zeros (zeroes at the end of the number) are there in $12!$?
34. King Arthur's 100 knights of the round table have been kidnapped! The evil wizard Gor has seated them at a circular table and numbered them 1 to 100. At the start of every second, starting with 1, Gor poisons every fifth knight (i.e. 1, 6, 11, and so on) with a poison that will kill the knight immediately. If there are less than 5 knights remaining, the one with the largest number is announced winner, and sent back to King Arthur. What is the number of this last knight?
35. Let $r_1, r_2, r_3, \dots, r_{100}$ be the roots of the polynomial $x^{100} - 12x + 3$. Find $r_1^{100} + r_2^{100} + \dots + r_{100}^{100}$.
36. Suppose you have a circle with center O and diameter 15. Point P is outside the circle and A is a point such that PA is a tangent. Extend A through O to get C on the circle and let B be the intersection of PC and the circle. $PB = 16$. Find PC .
37. Consider $x_1 + x_2 + \dots + x_{100}$, where $x_i \in \{0, 1, 2, 3, \dots, 99\}$ for all $i = 1, 2, 3, \dots, 100$. Find the number of solutions to $100 \mid x_1 + x_2 + \dots + x_{100}$.
38. How many solutions are there to the equation $x + y + z = \sqrt{2xy + 2yz + 2xz + 41}$ where x, y , and z are nonnegative integers?
39. In right triangle ABC , $BC = 24$ and $\angle B = 90^\circ$. A circle of radius 5 intersects sides AB, BC , and CA at P and Q , R and S , and T and U , respectively, such that $PQ = RS = TU = 6$. What is the length of leg AB ?
40. Haneul has a fair six-sided die, and she rolls the numbers a, b, c and d in that order. Let P be the polynomial of degree at most 3 such that $P(1) = a + b, P(2) = ab, P(3) = c + d$, and $P(4) = cd$. What is the expected value of $P(5)$?