Joe Holbrook Memorial Math Competition

4th Grade Solutions

October 14, 2018

1. $80 \text{ problems} \cdot \dfrac{1 \text{ minute}}{5 \text{ problems}} = \boxed{16}$ minutes.
2. We use PEMDAS: $2 \div (0+1) \cdot 8 = 2 \div 1 \cdot 8 = 2 \cdot 8 = \boxed{16}$.
3. There are 7 months with 31 days and 12 months in a year. Therefore, $\boxed{\dfrac{7}{12}}$ of the months have 31 days.
4. Jenn ate $\dfrac{2}{3} \cdot \dfrac{1}{2} = \dfrac{1}{3}$ of the cake. Therefore, Shalin and Jenn together ate $\dfrac{1}{3} + \dfrac{1}{3} = \dfrac{2}{3}$ of the cake. Thus, $1 - \dfrac{2}{3} = \boxed{\dfrac{1}{3}}$ of the cake remained.
5. Working backwards, we multiply 100 by 4 to find the total number of students: $100 \times 4 = \boxed{400}$.
6. Multiply to find the answer of $4 \cdot 10 \cdot 18 = \boxed{720}$ dollars.
7. Addition gives $1003 + 130 + 310 + 3001 = \boxed{4444}$.
8. Add the numbers directly to obtain the answer of $\boxed{2038.3818}$.
9. The number halfway between $\dfrac{1}{2}$ and $\dfrac{1}{4}$ is the average of $\dfrac{1}{2}$ and $\dfrac{1}{4}$, which is $\dfrac{\frac{1}{2} + \frac{1}{4}}{2} = \boxed{0.375}$.
10. On day n, the student solves $3n+1$ problems, since on day 1 she solved $3 \times 1 + 1 = 4$ problems and she solves three more problems than she did the day before. On day 12, she solves $3 \times 12 + 1 = \boxed{37}$ problems.
11. The number of games is equal to the number of ways to choose two teams from a set of 7, which is precisely $\dbinom{7}{2} = \boxed{21}$.
12. There are three primes between 1 and 6, inclusive: 2, 3, and 5. Therefore, the answer is $\dfrac{3}{6} = \boxed{\dfrac{1}{2}}$.
13. Haneul is 35 inches and has an additional 14 inches, so she is 49 inches. Andrew is $4.5 \times 12 = 54$ inches. $54 - 49 = \boxed{5}$.
14. Let x be Mikako's favorite positive number. Then,

    $$\sqrt{4x-11} = 3 \implies (\sqrt{4x-11})^2 = 3^2 \implies 4x - 11 = 9 \implies x = 5.$$

    Thus, Mikako's favorite positive number is $\boxed{5}$.
15. Each chopped off head results in 2 more heads. Therefore, the answer is $2 \cdot 7 + 20 = \boxed{34}$.
16. Kelvin needs \$100 and already has \$25, so he needs to earn \$75 from his job. Since he earns \$4 an hour, he will need to work at least $\dfrac{75}{4}$ hours. Since Kelvin only works a whole number of hours, round $\dfrac{75}{4}$ up to the nearest whole number. As a result, Kelvin will need to work $\boxed{19}$ more hours in order to afford his chicken nuggets.
17. Clearly, the net change in pizzas in each minute is 4 pizzas. So after 503 minutes, 2012 pizzas. After Herb makes another 7 pizzas in the $\boxed{504}$th minute, there are 2019 pizzas, and the goal is reached.

18. The factors of 2018 are 1, 2, 1009, and 2018. Thus, the sum of the divisors of 2018 is $1+2+1009+2018 = 3030$. The prime factorization of 2019 is $3^1 \cdot 673^1$. Thus, 2019 has 4 factors. The answer is $3030-4 = \boxed{3026}$.

19. The straight path from home to school is the hypotenuse of a right triangle with legs 8 and 15. Using the Pythagorean theorem, the hypotenuse has a length of $\sqrt{8^2+15^2} = 17$. If he walked west then south, Jason would walk 23 meters, so Jason would walk $23 - 17 = \boxed{6}$ meters less if he just walked in a straight path from home to school.

20. Since 2019 is odd, it cannot be the sum of two odd prime numbers. Therefore, $2019 = 2 + 2017$ is the only possibility, and there is exactly $\boxed{1}$ way.

21. We should only consider the ages of the children who are three years old or older. These ages are 3, 4, 7, 8, 9, 11, and 12. Through inspection, we conclude that the median of this set is $\boxed{8}$.

22. From the formula, we have
$$2 \star 1 = 5(2) + 3(1) + 1 \star 1 = 23 \implies 2 \star 2 = 5(2) + 3(2) + 2 \star 1 = \boxed{39}.$$

23. Initially, 180 Starbursts are pink and 420 are not pink. When one removes pink Starbursts only, the number of Starbursts which are not pink remains fixed at 420. If the number of not-pink Starbursts is to account for 80% of the bag after the removal, then there must be a total of $\frac{420}{0.8} = 525$ Starbursts at the end of the procedure. Hence, $\boxed{75}$ pink Starbursts must be removed.

24. $1000000^2 < 1234567^2 < 2000000^2$. Since both 1000000^2 and 2000000^2 have 13 digits, 1234567^2 also has $\boxed{13}$ digits.

25. Initially, it may appear that repeatedly multiplying by 2 at the outset gives the optimal number of operations. Using this strategy, the sequence is 2, 4, 8, 16, 32, 33, 34, 35. However, this strategy requires the addition of three 1s. Instead, we can add 1 before the last multiplication by 2, yielding 2, 4, 8, 16, 17, 34, 35. Thus, a minimum of $\boxed{6}$ operations are needed.

26. For $n \geq 2$, the probability that domino n is knocked over is $\frac{n}{n+1}$. Furthermore, for $n \geq 2$, domino n cannot be knocked over unless domino $n-1$ is knocked over. Therefore, the answer is $\frac{2}{3} \cdot \frac{3}{4} \cdots \frac{2018}{2019} = \boxed{\frac{2}{2019}}$.

27. The sum of Andrew's scores on the five tests he has taken is $94 \cdot 5 = 470$. In order to receive an average of 95 across all six tests, the sum of his scores on the six tests must be at least $6 \cdot 95 = 570$. Therefore, he must receive a score of $570 - 470 = \boxed{100}$ on his sixth test.

28. The least common multiple is $100 = 2^2 \cdot 5^2$. Since 4 has no factors of 5, x must be a multiple of $5^2 = 25$. We see that the only multiples of 25 that are factors of 100 are $25, 50, 100$. All of these work, so the sum is $\boxed{175}$.

29. Notice that the number of minutes it takes Andrew to write each problem forms an arithmetic sequence with first term 1 and common difference 1. Using the formula for the sum of an arithmetic sequence, we obtain an answer of $\frac{10 \cdot 11}{2} = \boxed{55}$ minutes. Alternate Solution: Note that Andrew spends a total of $1 + 2 + 3 + \ldots + 10$ minutes writing the ten problems. Since the sum of the first n positive integers is $\frac{n(n+1)}{2}$, the answer is $\frac{10 \cdot 11}{2} = 55$.

30. By the problem constraints, the numbers not divisible by 3 must form a contiguous block (the number of factors of 3 is nonincreasing). Similarly, the numbers with one factor of 3 and the numbers with two factors of 3 each form contiguous blocks. Finally, each block must be increasing, so we get the answer, $\boxed{(9, 3, 6, 12, 4, 8)}$.

31. First, note that the number of people in the room must be divisible by 40. Let the number of the people in the room equal to $40k$, for some positive integer k. Then exactly $25k$ people like chocolate and exactly $24k$ people like vanilla. This means at least $24k + 25k - 40k = 9k$ people like both (if any fewer liked both, then the total number of people would exceed $40k$). Thus, the smallest number of people who like both chocolate is $\boxed{9}$, achieved when $k = 1$. In this case, 9 people like both chocolate and vanilla, 16 only like chocolate, and 15 only like vanilla.

32. Recall that the area of an equilateral triangle with side-length s is $\frac{s^2\sqrt{3}}{4}$. Therefore, an equilateral triangle with side-length 20 has area $\frac{20^2\sqrt{3}}{4} = 100\sqrt{3}$. The square has area $13^2 = 169$. Since $\sqrt{3} > 1.7$, $100\sqrt{3} > 170 > 169$, and the equilateral triangle has a larger area. Thus, the answer is $\boxed{100\sqrt{3}}$. Alternatively, one could check that $100\sqrt{3} > 169$ by squaring both sides of this inequality to find $30000 > 28561$, which is true.

33. Sebastian's total purchase cost $10 - 4.61 = \$5.39$. Since each apple costs a whole number of cents, the number of apples he bought must be a factor of 539 which lies between 1 and 30. Since $539 = 7^2 \cdot 11$, the only such factors are 7 and 11. Therefore, the answer is $7 + 11 = \boxed{18}$.

34. First, we give a piece of candy to each child. Susan has 2 candies left to distribute. There are two cases:a Case 1: 2 children each receive 1 candy. There are $\binom{5}{2=10}$ ways. Case 2: 1 child gets 2 candies. There are 5 ways to select this child. Adding up, we find the answer of $5 + 10 = \boxed{15}$. Alternative Solution: By the Stars-and-Bars Formula, the answer is $\binom{7-1}{5-1=64=15}$.

35. The region that the lamb can wander in is a circle of radius 6 around the origin, with two circular cutouts of radius 6 from each of the two wolves. It can be noted that if you move the cut-out pieces properly, you can form a square with a diagonal of length 12. Therefore, the area of the region is $\boxed{72}$.

36. The sum $\frac{p}{q} + \frac{q}{p}$ can be rewritten as $\frac{p^2+q^2}{pq} = \frac{(p+q)^2 - 2pq}{pq}$. The roots are 5 and 7. Plugging these values into the above fraction yields the answer of

$$\frac{(p+q)^2 - 2pq}{pq} = \frac{144 - 70}{35} = \boxed{\frac{74}{35}}.$$

37. Let t be the amount of time it takes (in hours) for the trains to meet. Then $360 = 60t + 60t \implies t = \frac{360 \text{ miles}}{120 \text{ miles/hour}} = 3$ hours. Therefore, the crow flies for 3 hours before the trains meet. Hence, the crow travels a total distance of 40 miles/hour $\times$ 3 hours $= \boxed{120 \text{ miles}}$ before the trains meet.

38. The value of ϕ is irrelevant! The given expression can be rewritten as $\frac{(x+\phi)^2 + 2017}{2018}$, which has a minimum value of $\boxed{\frac{2017}{2018}}$ at $x = -\phi$.

39. This means that the constants can be considered identical. Now we have 4 "e"s and 5 consonants. There are $\binom{9}{4}$ ways to put the "e"s in their places. As the consonants are in either in alphabetical order or reverse we need to multiple by 2 to consider the different cases. This yields a total of $\binom{9}{4} \times 2 = \boxed{252}$ rearrangements.

40. All of the terms are of the form $\frac{1}{n(n+2)}$. This looks like a telescoping sum problem, and so each of the terms is probably similar to $\frac{1}{n} - \frac{1}{n+2}$ so that everything cancels out. Sure enough, $\frac{1}{n} - \frac{1}{n+2} = \frac{2}{n(n+2)}$, and so $\frac{1}{n(n+2)} = \frac{1}{2}\left(\frac{1}{n} - \frac{1}{n+2}\right)$. Rewriting each of the terms in this way, the sum becomes $\frac{1}{2}\left[\left(\frac{1}{1} - \frac{1}{3}\right) + \left(\frac{1}{3} + \frac{1}{5}\right) + \cdots + \left(\frac{1}{17} - \frac{1}{19}\right)\right]$. This sum telescopes to $\frac{1}{2}\left(1 - \frac{1}{19}\right) = \boxed{\frac{9}{19}}$, which is the answer.