

Joe Holbrook Memorial Math Competition

5th Grade

October 18, 2020

General Rules

- You will have **75 minutes** to solve **40 questions**. Your score is the number of correct answers.
- Only answers recorded on the answer sheet will be graded.
- This is an individual test. Anyone caught communicating with another student will be removed from the exam.
- Scores will be posted on the website. Please do not forget your ID number, as that will be the sole means of identification for the scores.
- You may not use the following aids:
 - Calculator or other computing device
 - Compass
 - Protractor
 - Ruler or straightedge

In addition, you must use the scrap paper supplied by the proctors.

Other Notes

- Write legibly. If the graders cannot read your answer, you will be given no credit for that question.
- Fractions should be written in lowest terms. Please convert all mixed numbers into improper fractions.
- For constants such as e or π , do not approximate your answer: for example, if the answer to a question is 7π , then you should not write 22 or 21.99.
- You do not need to write units in your answers.
- Rationalize all denominators. In addition, numbers within a square root must be squarefree, e.g. $\sqrt{63}$ should be written as $3\sqrt{7}$.
- Ties will be broken by the number of correct responses to questions 31 through 40. Further ties will be broken by the number of correct responses in the last five questions.

1. How many cents is 4 quarters, 3 dimes, 2 nickels, and 1 penny?
2. A guy who is 76 inches tall with his shoes on stands on a 12 inch tall bench, but takes off his 2-inch elevated shoes. How far off is the top of his head from the ground now?
3. If 4 scores and 7 years is equivalent to 87 years, how many scores is 2020 years?
4. Alicia is learning the Korean alphabet. If there are 14 basic consonants and 10 basic vowels, how many "basic letters" can she create? (A "basic letter" consists of one consonant followed by one vowel.)
5. Alex types at 50 words per minute (wpm) and Yoland types at 120 wpm. If they both start typing a 3000 word essay, how many more minutes will it take Alex to finish?
6. Erez forgets things at a rate of 5 things per minute, and he doesn't learn any new things. If his brain will be empty in exactly 2.5 hours, how many things are in his brain right now?
7. In Noah's backyard, a bird chirps once every 10 seconds, a frog croaks once every 15 seconds, and his dog barks once every 4 seconds. If the three animals just made sounds simultaneously, how many seconds later will they again make a sound in unison?
8. In a very small dictionary, there are 500 words and they all consist of either 3, 4, 5, or 6 letters. There are 27 three letter words, 166 five letter words, and 92 six letter words. If a word is selected at random, and the probability it has four letters can be expressed as $\frac{a}{b}$ in lowest terms, what is $a + b$?
9. Erik's code doesn't work, and every time an error message pops up, there is a $\frac{2}{3}$ probability that he throws his computer out the window. If the probability that he makes it through 6 error messages without damaging his computer can be expressed as $\frac{a}{b}$ in lowest terms, what is $a + b$?
10. When using Zoom, each participant's face appears in a 2 inch by 3 inch box. You have a projector screen that is 9 feet by 16 feet. Making sure that everyone's box is fully inside the screen, what is the greatest number of participants you can see at once without overlapping?
11. On planet Gloop, they use the Gankine temperature scale. To convert from Gankine to Farenheit, you multiply the Gankine temperature by $\frac{23}{2}$ and add 4. If today is 4 Gankine degrees hotter than yesterday, what's the temperature change in Farenheit?
12. A fish tank in the shape of a rectangular prism has a base length of 24 inches and a base width of 12 inches. Currently, water fills up the tank to a height of 10 inches. While cleaning, 864 cubic inches of water is removed. By how many inches did the height of the water decrease?
13. In a class of 10 students, 7 students own a dog, 6 students own a cat, and 1 student owns neither. How many students have both a dog and a cat?
14. Super Cool Jessica is flipping an unfair coin that lands heads with probability $\frac{2}{3}$ and tails with probability $\frac{1}{3}$. If she flips the coin twice, the probability that she receives exactly one heads and one tails is $\frac{p}{q}$ where $GCD(p, q) = 1$. What is $p + q$?
15. For every day that Jess practices the Rubik's Cube, her time will decrease by 3 seconds and for every day she doesn't practice, her time will increase by 5 seconds. She decides to train during the month of October, hoping to decrease her current time of 60 seconds to less than 10 seconds. How many days of October at most can Jess not practice and still reach her goal?
16. Albert's average for 5 quizzes is an 88 right now. If he gets a 95 on his next quiz and his teacher also discards his lowest grade out of the 6 quizzes, his average increases to a 93. What was Albert's lowest quiz grade?
17. Jaiden the Baker makes cookie batter. In this batter, the ratio of milk to flour is 2:5, the ratio of flour to sugar is 7:2, the ratio of sugar to baking powder is 3:1, and lastly, the ratio of baking powder to butter is 2:9. If there are 135 grams of butter in the batter and all the ratios are in terms of grams, how many grams of milk does the batter have?
18. Yul the Cül likes to be cool in the summer, so she eats a lot of watermelon. She has a perfectly spherical watermelon of diameter 12 in., which consists of a sphere of flesh surrounded by a rind that has a uniform thickness of 1 in. If the volume of the surrounding rind can be written in the form $\frac{a\pi}{b}$ cm³ for relatively prime a, b , find $a + b$.

19. When Betsy the Cat is standing on table A, the top of her head is 3 feet above the top of table B. When she's standing on table B, the top of her head is 4 inches below the top of table A. How tall is Betsy (in inches)? (Assume that the thickness of both tables' surfaces is zero).
20. 200 points are equally spaced on the circumference of a circle. How many squares can be formed with 4 of the 200 points as vertices?
21. Find the largest 5-digit number such that all the two digit numbers formed by pairing adjacent digits in the 5-digit number will be prime. The primes created in this way need not be unique.
22. Find the largest integer n such that n is divisible by 5, the sum of the digits of n is divisible by 6, and $n < 1000$.
23. There are 6 chairs in a row and 3 students, and no student wants to sit next to another student. How many ways can the students order themselves to sit?
24. Dan is juggling a basketball, a tennis ball, a soccer ball, a ping-pong ball, and a bowling ball. He drops at least one item, but not all of them. How many possible sequences of the balls hitting the floor are there? For example, some possible sequences are a tennis ball; a basketball and then a tennis ball; or a ping-pong ball, a bowling ball, and then a basketball.
25. Grammarly Greg is writing a dictionary. The pages of his dictionary are numbered from 1 to n , however he forgot to number a page. Let m denote the number for the page he forgot to number. Given that the sum of the page numbers, excluding the one he forgot to number, is 200. Find $n + m$.
26. Galactic Greg is searching for the secret to astronomy! Once he finds the secret, he stops searching, and given he has not found it before the probability that he finds this secret on any given day n is $\frac{1}{2^n}$. The probability that Greg will find the secret before day 9 is $\frac{p}{q}$ where $(p, q) = 1$. What is the $p + q$?
27. Consider parallelogram $ABCD$. Let E be on AB such that $AE : EB = 3 : 1$. Line DE intersects line CB at F . If $EF = 5$, what is DE ?
28. In a little town, people speak in either base three or base five and some people are fluent in both bases. However, they may only use one base in each sentence. A reporter interviewed two residents to figure out how many people lived in the town. The first person said "101 people speak base 12 and 20 people speak both" and the second person said "20 people speak base 3 and 11 speak both bases". How many people live in this little town?
29. Let n be the least positive integer such that the product of the digits of the number is $7!$. Compute the sum of the digits of n . Note that $n! = n \cdot (n - 1) \cdot \dots \cdot 2 \cdot 1$.
30. Alicia is eating dried mangoes. The probability that the piece of mango is sweet is $\frac{2}{5}$ and the probability that it's sour is $\frac{3}{5}$. If the one she eats is sweet, she will eat another one. If the one she eats is sour, she will stop eating dried mangoes. If the expected number of dried mango pieces she will eat is in the form of $\frac{m}{n}$ and m, n are relatively prime, compute $m + n$.
31. Yul has a special calculator that can do four operations: multiply the input by 3, multiply the input by 5, subtract 4 from the input, or subtract 6 from the input. A number is called *good* if the number 5 cannot be made, after any number of operations. However, the number must always stay positive. For example, 7 is not *good* because the following operations can be done to make 5: $-6, \times 5$. What is the sum of the first three *good* numbers that are greater than 100.
32. A fly and a mantis are sitting on the ends of the minute and hour hands of a clock respectively. The hour hand is 5 cm long and the minute hand is 10 cm long. When the mantis is within $5\sqrt{3}$ cm of the fly, it can reach out and grab it. If it is 3 o'clock right now, how long will it be, to the nearest minute, until the mantis can reach out and grab the fly?
33. Take the integers from 1 to 9 and place them in the 9 cells of a 3 by 3 grid, one per cell. Given that for every pair of adjacent cells, the product of the numbers in them isn't a perfect square, and that every 2 by 2 grid that's part of the 3 by 3 grid has exactly one prime in it, how many ways were there to arrange the numbers?

34. The math team captain wants to cut her 2-D circular watermelon to share with the 137 other people on the math team. If she can only make straight cuts that are chords of the circle, what is the smallest number of cuts she could make so that everyone on the math team gets a piece?
35. A wild traus (a lake-inhabiting animal) sitting at a corner of a unit cube wants to make its way to the vertex furthest from it by only crawling on the surface of the cube. However, two of the six sides of the cube are chosen randomly and set on fire, restricting the traus from crawling on them. The expected value for the length of the shortest path the traus can take to reach the desired vertex is a , what is the nearest integer to $8a$?
36. Let S be the set of all positive integers whose largest prime factor is 11. The sum of the reciprocals of all the elements in S is $\frac{m}{n}$ for relatively prime positive integers m and n . Compute $m + n$.
37. In trapezoid $ABCD$, $AB \parallel CD$ and $CD > AB$. Also, $AB = AC = AD = 5$ and $BC = 4$. If the length of BD is $a\sqrt{b}$, what is $a + b$?
38. The probability that the number of 7's in the prime factorization of a randomly selected positive integer is divisible by 3 is equal to $\frac{x}{y}$ for x, y relatively prime. (Note that 0 is divisible by 3). Find $x + y$.
39. Points E and F are randomly picked on the perimeter of rectangle $ABCD$, with $AB = CD = 20$ and $BC = DA = 37$. The probability that line segments AE and CF intersect can be written in the form $\frac{m}{n}$, for relatively prime positive integers m and n . Compute $m + n$.
40. Justin is playing a game. There is a machine that has four identical balls, and will randomly select four squares on a six by six grid to put these balls on. Call an orientation "laughable" if no two balls lie in the same row or column, and no ball lies diagonally to the right of and above another ball. Justin wins the game if the orientation is "laughable." The probability that Justin wins can be expressed as $\frac{m}{n}$, where m and n are coprime. Find $m + \frac{n}{11}$.