Joe Holbrook Memorial Math Competition

7th Grade

October 18, 2020

General Rules

- You will have **75 minutes** to solve **40 questions**. Your score is the number of correct answers.
- Only answers recorded on the appropriate Google Form will be graded.
- You are to remain visible to your proctor at all times. Please have your video camera on during the exam.
- This is an individual test. Anyone caught communicating with another student or using technology in an inappropriate way will be removed from the exam.
- Scores will be posted on the website. Please do not forget your ID number, as that will be the sole means of identification for the scores.
- You may not use the following aids:
  - Calculator or other computing device
  - Compass
  - Protractor
  - Ruler or straightedge

Other Notes

- All answers are integers. Please enter them with no spaces in between into the Google Form. For a negative integer, please enter -7 not $-\ 7$.
- Do not include commas in your answers. For example, the number one thousand is to be entered 1000 not $1,000$.
- You must not write units in your answers.
- Ties will be broken by the number of correct responses to questions 31 through 40. Further ties will be broken by the number of correct responses in the last five questions.

1. Compute $\dfrac{5^2 \times 5^2}{5^2} + \dfrac{5^2 + 5^2}{5^2}$.

2. Jessica writes a letter to Vivian. It takes 35 minutes to write, 2 minutes to address and seal, 4.2 hours for the mailman to pick it up, 2.5 days for the letter to be delivered, 1.6 hours for Vivian to realize it is in her mailbox, and 4 minutes for Vivian to read the whole thing. How many minutes has it been since Jessica first started writing the letter, given no breaks in the described process?

3. On Halloween, the sun sets at 5:52 pm. It will take an additional 87 minutes for full darkness to settle in. If Frank needs to be back when it turns completely dark and starts trick-or-treating at 4:37 pm, how many minutes will he have to trick-or-treat?

4. A computer wallpaper has a resolution of 1920 pixels wide by 1080 pixels tall. If the dimensions of the image were scaled down to $\frac{1}{6}$ of its original size, it would have a width of a pixels and height of b pixels. What is $a + b$?

5. Pratham has stuffed turtles and mallard ducks. He counts 17 heads and 60 legs. How many turtles does he have?

6. Bettie has a lot of chocolate from trick-or-treating. She gives one-third of her chocolates to Andy. Andy eats half of the candy he received and has 6 left. Bettie's little brother then steals half of Bettie's remaining candy. Finally, Bettie eats 2 chocolates. How many chocolates does Bettie have now?

7. Triangle A and Triangle B have the same area. Triangle A has a side of length 12 with the height to that side being 9. Triangle B has a side of length 18. What is the length of the height to that side of Triangle B?

8. 2020 is a year that can be written by using 2 distinct digits exactly 2 times. When is the next year this will happen?

9. Kevin refuses to go outside if the temperature has less than 5 factors. The temperatures this week were 61, 62, 63, 64, 65, 66, and 67 degrees. How many of these days did Kevin stay home?

10. Vivian is running for vice president. She has \$20 dollars that can be used to buy votes. There are 5 districts that she can buy votes from; each represented by an ordered pair (a, b) where a is how much she needs to pay to win the district and b is how many votes the district will provide her. Giving less than \$$a$ to any district will not give Vivian any votes, and each district can only provide her a maximum of b votes. Given that the 5 districts have ordered pairs: $(20, 20); (15, 20); (5, 10); (4, 20); (10, 20)$, what is the maximum number of votes she can earn?

11. David is renovating his rectangular bathroom, whose sides are a whole number of feet. At the last minute, David decided to expand his bathroom's length by 2 feet and width by 3 feet. If his bathroom's new area is 45 square feet, what is the largest area the bathroom could have had before the expansion?

12. Yul plays Jaiden in Word Hunt every day. Whenever they play, Yul scores 1,500 and Jaiden scores 15,000. Frustrated by her losing, she decides to practice. On her n-th hour, she gets better by a score $100n$. ie, after one hour, her score increases by 100 to 1600, after two hours her score increases by another 200 to 1800. How many hours will she need to practice to finally beat Jaiden?

13. What is the sum of the common factors of 2020, 606, and 2222?

14. Isaac and Renee were talking about their parents. Isaac asked Renee to guess their ages, so Renee asked for some hints. Isaac said, "my parents' ages each have either 3 or 4 factors." Renee said, "That's not enough information," so Isaac added, "My parents are both older than 20 and younger than 50, and their ages end in 9." Renee said again, "still not enough information," to which Isaac responded "My dad's at least one year older than my mom." What is the sum of the ages of Isaac's parents?

15. In a Zoom lecture, 24 people have their cameras on and 10 people have their microphones on. If a person's camera is on, that means they are dressed up. 37 people are dressed up, and of those, 5 have their microphones on. Exactly 2 people have their camera and microphone on and are dressed up. There are 80 participants total. How many have their camera and microphone off, and are not dressed up?

16. Emma likes to split squares into numerous rectangular cells by drawing vertical and horizontal lines parallel to the sides of the square, that start from one edge of the square, and end at the opposite edge of the square. If it takes her 48 seconds to split a square into 25 cells, what is the largest amount of time, in seconds, it could take her to split another, congruent square into 18 cells? Assume she draws all of the lines at the same, constant speed.

17. There are 9 friends going on a trip and they booked three cabins with capacities of 2, 3, and 4 people. How many ways are there for the 9 friends to stay at these three cabins if three certain friends, namely A, B, and C, are to stay in different cabins?

18. Suppose m is a two-digit positive integer such that $8^{-1} \pmod{m}$ exists and $8^{-1} \equiv 8^2 \pmod{m}$. What is m?

19. Albert, Bertha, Clyde and Dale are building a house. Each works at a constant rate. If Albert worked twice as fast, he and Bertha could build the house in 4 days. If Clyde worked twice as fast, he and Bertha could build the house in 8 days. All working together (and at their normal speeds), the four people can finish the house in 2 days. To the nearest day, how long would it take for Dale to finish the house on his own?

20. There exists a nondegenerate triangle ABC with integer side lengths such that BD is an angle bisector, where D lies on $\overline{AC}$. BD cuts AC into pieces of lengths 7 and 3. Find the minimum possible perimeter of $\triangle ABC$.

21. Jim has 15 identical apples he wants to distribute amongst his 5 friends. However, his friends get easily jealous, and will get jealous if someone else gets more than twice as many apples as they got. In how many wants can Jim distribute his apples such that no one gets jealous?

22. Jaiden and Yul play 8 Ball every day. Every time Jaiden wins, he gets very happy, so he twirls 8 times. Every time they tie, he twirls 4 times. Every time Yul wins, she twirls 5 times, and when they tie, she twirls 3 times. What is the greatest number that cannot be the sum of the number of times Jaiden and Yul have twirled?

23. A positive integer divisor of 10! is chosen at random. The probability that it is a perfect square but not a perfect cube is p/q where p/q is a fractional in lowest terms. What is $p + q$?

24. In trapezoid $ABCD$, segments AB and CD are parallel such that $CD > AB$. The measure of angle B is twice the measure of angle D, and $AB = 4$ and $BC = 10$. What is CD?

25. A group of n friends are playing a card game with a pack of 126 cards. After the cards are dealt out, everyone has at least one card, but the players realize that exactly one half of them each have one more card than each of the other half. What is the sum of the possible values of n?

26. A square of side length y is wholly inscribed within a square of side length x where x and y are integers. If the area between the two squares is 2020, find the minimum possible value of x.

27. How many permutations of the numbers 1, 2, 3, 4, and 5 are there such that no three consecutive numbers in the permutation form an arithmetic series?

28. Call a word formed from the letters a, b, and c *mayan* if between any two a's (not necessarily adjacent) there's a b, between any two b's there's a c, and between any two c's there's an a. How many mayan words of length 2020 start with abc?

29. For how many values of c in the interval $[0, 1000]$ does the equation

$$7\lfloor x \rfloor + 2\lceil x \rceil = c$$

have a solution for x?

30. Compute the sum of all positive integers that are equal to 105 plus their largest prime factor.

31. A cube of side length $\frac{7}{3}$ is dropped into a random place on the 3D lattice in the coordinate space (a coordinate plane with an extra dimension), with the sides of the square parallel/perpendicular to the axes. The expected number of lattice points (points with integer coordinates) in the interior of the cube is $\frac{a}{b}$ in simplest terms, what's $a + b$?

32. Let triangle ABC be a triangle such that $\angle ABC = 120°$. Let D be a point on $\bar{AC}$ such that $\bar{BD}$ bisects $\angle ABC$. If the ratio of the length of $\bar{AB} : \bar{BC} = 2 : 1$, let the ratio of the length of $\bar{BD} : \bar{BC} = m : n$, where m and n are relatively prime integers. Find $m + n$.

33. A number with an even amount of digits is called a "sandwich number" if the first half of the digits are in strictly decreasing order and the second half of the digits are in strictly increasing order. For example, 987359 is a sandwich number while 988778 is not. If $\frac{m}{n}$ of all six-digit sandwich numbers are palindromes (numbers that read the same backward and forwards) and m and n are relatively prime positive integers, what is $m+n$?

34. Chrissy's ice cream shop serves big scoops (30 grams each) and small scoops (10 grams each). One day, she gets a very strange order from a customer. The customer wants 1000 grams of ice cream in total but does not want the number of small scoops to be divisible by the number of big scoops. If Chrissy cannot take out scoops and can only add in a whole number of scoops, how many ways can she fulfill this odd order (order of scoops does not matter)?

35. Two boxes are stacked on top of each other, against a vertical wall. The bottom box comes out 12 inches, is 3 inches wide and 8 inches tall. The top box comes out 4 inches, is 3 inches wide and 6 inches tall. A 3-inch wide plank is secured so that it leans against these two boxes and the wall. Then the boxes are removed, while the plank stays still. What is the volume of the largest box that can fit under this plank, without poking out either side?

36. In a certain family, there is one father, one mother, and some children (some boys and some girls). The average number of male relatives for a person in the family (where the relatives are also in the family) is 6.75. If there are m males and f females in the family, compute the value of $10f + m$.

37. Yul is addicted to her phone. 20 minutes after she wakes up, she has a $\frac{1}{4}$ probability of picking up her phone. At every 20 minute interval, if she picked up her phone 20 minutes ago, the probability resets to $\frac{1}{4}$. Otherwise, the probability that she picks up her phone doubles. What is the expected number of minutes it takes for Yul to pick up the phone 8 times?

38. Consider triangle ABC with circumcenter O, such that AO is parallel to BC. If the circumradius of ABC has length 4 and side BC has length 5, then compute the value of AC^2.

39. Compute the smallest positive integer b for which there exist positive integers A and B less than b such that the value of BA_{2b} (the two digit number BA in base $2b$) is 9 times the value of AB_b (the two digit number AB in base b).

40. Consider trapezoid $ABCD$ with $AB \parallel CD$ and $AB < CD$. The angle bisector of $\angle BCD$ and the angle bisector of $\angle ADC$ intersect line AB at points E and F respectively, such that quadrilateral $DEFC$ is a parallelogram. The two angle bisectors also intersect each other at point P. Given that $AD = 10$, $DP = 8$, and that the area of trapezoid $ABCD$ is 216, compute the length of BC.